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A REVIEW OF THE NORMAL MODE INITIALIZATION METHOD

by

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C O N T E N T S

1. INTRODUCTION

The weather forecasting equations are four-dimensional, non-linear partial differential equations. One of the characteristics peculiar to partial differential equations is that the initial and/or boundary conditions are intimately connected with the equations, and cannot be given independently as in the case of ordinary differential equations.
In the field of linear differential equation theory, it is well known that for equations of elliptic type a boundary value problem must be posed. The Cauchy problem for these equations is not well posed. The four-dimensional non-linear partial differential equations used in weather prediction are far more complicated. How to treat the initial data and make the forecasting problem mathematically well-posed are by no means simple things. The initialization problem is concerned with specifying initial conditions which are meteorologically "well-posed" in that they lead to a meteorologically realistic result in the time integration, see Tribbia (1980).

This is a very important point and it has taken quite a long time in the development of numerical weather prediction to realize it. The lack of success of the first numerical weather prediction by Richardson in 1922, to a great extent, was rooted in the disregard of this principle. So, indeed, the specification of the initial state is one of the most important aspects of deterministic forecasting by numerical means, see Daley (1980) and Ballish (1980).

Richardson's failure had called attention to the need to get rid of the oscillations of high frequencies in a forecasting model. Although such oscillations correspond to some possible solutions of the dynamic equations for atmospheric motion, they are at the most a very minor component of the observational weather data. The proper calculation of true features requires a very, very high accuracy in both observation and computation. The required degree of accuracy is too high to be reached practically, at least, at the present time and in the near future. Inevitably, observational and analysis errors can produce a large component of such oscillations in the initial state, which can then grow rapidly in the procedure of time integration, and contaminate the weather forecasts. These high frequency oscillations have been called meteorological noise and they have proven themselves to be a cancer for numerical weather prediction.

Since Rossby derived and published his vorticity formula in 1939, the main efforts had been concentrated on the modifications of the dynamical equations,so that they no longer permit the noise component to be a solution. Such modified equations constitute the bases for quasi-geostrophic and quasi-non-divergent models. These models have dominated numerical weather prediction up to the beginning of the sixties.

Increases in observational data, progress in computational facilities, and comprehensive understanding of the various physical factors influencing the behaviour of the atmosphere - all these taken together from the sixties - stimulated investigations anew to use the primitive equations. Since an unmodified primitive equation model is kept in use, a key point is to rid the initial state of unwanted noise.

The thermal field and mass field are related by the hydrostatic equation. This relationship is well maintained even in the radiosonde data. It is, however, more difficult to balance the initial mass and wind fields. The required balance is quite subtle, and a small imbalance in the initial state will cause inertia-gravity oscillations with amplitudes much greater than those observed in the atmosphere, and eventually contaminate the forecast, see Wiin-Nielsen (1978).

In the early stages of applying the primitive equations in weather prediction, the initialization borrowed the relationships from quasi-geostrophic or quasi-nondivergent models. This means, to some extent, that the initialization procedurewas torn away from the dynamic equations used in forecasting. Therefore, some incompatibility may still exist between initial conditions (data) and differential equations. This deficiency was overcome by dynamical initialization. The dynamical initialization method, however, involves forward and backward integration, and is time consuming, see Bengtsson (1975).

2. LINEAR NORMAL MODE METHOD

Recently a relatively new method, based upon normal mode solutions to linearized systems of the forecasting equations, has broken fresh ground in the initialization field. This method was first used by Flattery (1970) in his Hough function analysis. The variation of the meteorological elements in the vertical direction (along the p-coordinate) was approximated by empirical orthogonal functions. The first seven terms in the empirical orthogonal expansion were used to describe the initial data on the 12 isobaric surfaces from 1000 mb to 50 mb. Then the three-dimensional weather forecasting problem was reduced to a two-dimensional one in the horizontal. In this case the governing equations are equivalent to the shallow water equations, which are then linearized about a basic state at rest. Their normal modes are Hough functions, which were first introduced in tidal research. In the west-east direction all the waves with wave lengths less than 15° are neglected. In the north-south direction from northern to southern poles only the first 24 terms are retained. That is to say, the atmospheric data are expanded in terms of the same Rossby modes for all levels. The use of only the Rossby modes made the initial fields free of gravity oscillations.

Flattery's work has been used in operational routine analyses at the NMC, USA. He succeeded in finding an analytical solution to the linearized forecasting equations at the cost of using a basic state at rest and empirical orthogonal functions in the vertical. In a more general approach the linear normal mode initialization on a sphere was studied by Wiin-Nielsen (1979b).

A linear normal mode filtering method and a general approach to find the normal modes for a numerical model was proposed by Dickinson and Williamson (1972). Their approach was illustrated by application to a global finite difference two-layer model. The basic state around which the shallow water equations are linearized is also at rest and the vertical dependence is separated from the horizontal. Along each latitude the data are expanded in truncated Fourier series. Along each longitude the derivatives are approximated by centred differences with a special treatment at the polar points. Normal modes obtained in their numerical form are compared with the analytical solutions and are classified as westward propagating Rossby waves, westward propagating inertia-gravity waves and eastward propagating inertia-gravity waves.

The number of normal modes for each zonal wavenumber is equal to the number of the grid points. For each kind of wave the normal modes are ordered according to the zero crossings between two poles, and so each mode has its own index. It is well-known that the fewer grid points contained in a wave length, the less accurately the wave can be described. When the normal mode index is equal to or more than half the number of grid points, this mode cannot be represented properly. Dickinson and Williamson (1972) called these kinds of modes "computational". But one must remember that no matter how many grid points are taken, all the numerically obtained normal modes taken together are always still in number less than the number of the normal modes included in differential equations. The number of normal modes is a countable infinity for differential equations.

In the linear normal mode initialization the observational data are expanded in terms of the complete set of normal modes, and then the expansion coefficients of unwanted "computational" and gravity modes are set to zero. These unwanted modes have a much larger amplitude in the real synoptical data than in climate simulation studies. Williamson (1976) had applied this principle to a global barotropic grid point model. This method déos reduce the unrealistic large-amplitude, high-frequency oscillations, which occur during the initial states of the forecasts with a conventional initialization procedure. But these unwanted high-frequency oscillations can be regenerated during the time integration due to non-linear interactions. This deficiency to a significant extent may be eliminated by non-linear normal mode initialization.

3. NON-LINEAR NORMAL MODE METHOD

Non-linear normal mode initialization is defined as a filtering procedure for which the time derivatives of the gravity mode coefficients are set equal to zero, while the gravity mode coefficients are modified in such a way that the linear contribution to the tendency of each coefficient (which depends only on the coefficient itself) compensates the contribution from the non-linear interactions between all the modes.

The non-linear normal mode initialization procedures were proposed by Machenhauer (1977) and Baer (1977). For accomplishing the procedure, an iterative scheme was proposed, and after only two or three iterations the gravity oscillations in practice are eliminated almost completely.

To apply the normal mode technique to a baroclinic primitive equation model, it is preferable to separate the vertical variables from the horizontal ones. This can easily be done in the absence of mountains and with the assumption that the basic state of the atmosphere is at rest. For the NCAR GCM with z coordinate the normal mode decomposition was carried out by Williamson and Dickinson (1976).

Normal mode initialization has also been put into the multi-level grid point model in ECMWF, see Temperton and Williamson (1979). The basic state is a function of sigma and is assumed to be a hydrostatic mean state at rest. All the non-linear terms are put in the right-hand side of each equation and combined into one term. The vertical variation was discretized at the beginning. Let us denote the number of levels by n. Then an n-dimensional vector is defined for each variable. The components of the vector are the variables on various levels. There are five such vector variables: zonal and meridional wind velocities, geopotential, temperature, and surface pressure. Then five vector differential equations are used: two equations of motion, thermodynamic equation, equation of continuity and hydrostatic equation. A new combined vector variable is defined such that its derivative gives the linearized horizontal pressure gradient. Then the last three equations are combined into one. The coefficient matrix on the left-hand side of this combined vector equation, in general, must have non-zero off-diagonal elements. In order to separate variables and determine the vertical structure of the normal modes, this matrix is diagonalized. That is, it is right multiplied by a matrix and left multiplied by its inverse matrix. Each column of this multiplying matrix is referred to as a vertical mode. Each entry in the diagonalized matrix is the corresponding eigenvalue, and enters into the shallow water equation as the equivalent geopotential depth. With these transformations the original system decouples into n independent systems, each having the form of the shallow water equations.

Despite the use of the combined vector variable, the vertical normal modes really are found in sigma coordinates, e.g., they properly take into account the boundary conditions $\dot{\sigma} = 0$ at $\sigma = 0$ and 1. There is an implicit assumption of no mountains, because otherwise the basic state would change in time due to non-zero pressure gradients. So, in this case, the assumed basic state (no motion, and temperature is a function of sigma only) is inconsistent, except in the special case of an isothermal atmosphere (temperature is independent of sigma).

A topographic experiment was performed by Daley (1979), using an isothermal basic state for the initialization process. It was found that in this case, the initially adjusted fields and the time behaviour of the subsequent model integration were very similar to the case where basic state temperature was allowed to vary with sigma. The vertical motion fields produced by the initialization procedure are consistent with the inclusion of topography, i.e. upward (downward) on the upstream (downstream) side of mountain barriers.

The non-linear normal mode initialization routine for the global spectral model at ECMWF was implemented by Andersen (1977). The technique used in separating variables and in finding vertical modes is the same as for the grid point model. The non-linear terms during the separation are considered as constants. The horizontal dependencies are expressed by a truncated spectral expansion in spherical harmonics with a triangular truncation. The results of the experiment performed with the 9-level ECMWF spectral model agree with those of that performed at the University of Copenhagen with a 5-level hemispheric spectral model of rhomboidal truncation.

The first set of one day forecasts performed with the normal mode initialization showed a substantial improvement judged by the standard deviation, as well as from the S_1 skill score. This is especially true in the low-latitudes - equatorward of 30^o, where the uninitialized case shows the presence of gravity waves with unrealistic height variations. These numerical experiments are consistent with Wiin-Nielsen's (1979a) comparative study. Using an idealized initial field consisting of only low order modes, Wiin-Nielsen reached the conclusion that the normal mode initialization should be particularly important at low latitudes on the large scale. One possible way to further improve initialization at low latitudes is to include zonal wind and its shear into the basic state, see Zhou, Di and Du (1980).

A comparative study with the normal mode initialization experiments also showed that nearly the same results are obtained if this filtering method is applied to the one day forecasts instead of the initial data. This indicates that the improvement is a result of the smoothing due to the initial filtering rather than an improvement in the forecasts. These facts imply that the gravity oscillations and/or the computational modes neither grow nor influence significantly the meteorological modes during the one day integration. This conclusion has been reached by all normal mode initialization without considering diabatic processes. It might not be valid when the model includes forcing terms such as the release of latent heat which are strongly influenced by the gravity waves. Inclusion of these terms into the non-linear normal mode initialization deserves special investigation, as it violates the fundamental assumption made in the non-linear correction procedure. This assumption is that the non-linear terms vary in time much more slowly than the modes being modified, and thus may be considered as nearly constant.

This fundamental assumption can also be violated when applied to multiple level models. The number of vertical modes is equal to the number of levels used in the model. The gravity modes corresponding to the small equivalent depths have very low frequencies. When all the vertical modes are included, the time scales of the non-linear terms are no longer greater than those of the modes being modified. The experience in ECMWF suggests that the optimum version is to make two iterations with five vertical modes included for both 9-level and 15-level models, see Temperton and Williamson (1979).

4. GENERALIZATION

The above mentioned fundamental assumption in non-linear normal mode initialization is always valid in problems with low Rossby number, a condition satisfied for many atmospheric flow regimes. Therefore, it is naturally possible to examine this problem from a more general point of view, using the method of expansion on small parameters. This quite different but more general approach was proposed by Baer(1977) and Baer and Tribbia (1977). They non-dimensionalized the forecasting equations such that non-linear terms seemed to be of order of the Rossby number which is assumed to be small. The coefficients of the eigenvectors of the linear system are expanded in a series of powers of the Rossby number. Using both a fast (external gravity mode) and a slow (inertia motion) time scale, the resulting equations were solved to different orders of the Rossby number. The required initial conditions are established to any order in Rossby number such that no gravity-inertia oscillations will occur to that order. Thus, zero and first order solutions are equivalent to the linear normal mode initialization. The second order solutions are similar to non-linear normal mode initialization,which give the adjustment necessary to remove high frequency oscillations from the non-linear terms in the forecasting equations. The scheme has been satisfactorily tested on a simple shallow water model and shows significant reduction of gravitational oscillations with the second-order adjustment.

This problem was also investigated by Leith (1979) in a model wave vector space. Following Leith (1979) we will call the set of Rossby modes Y and the set of gravity modes Z. If we now project the equations of the assimilating model onto its normal modes, we will obtain a set of ordinary differential equations, which we write symbolically as

$$\dot{Z} = -i\Lambda_Z Z + N_Z(Z,Y), \tag{1}$$

$$\dot{Y} = -i\Lambda_Y Y + N_Y(Z,Y), \tag{2}$$

where Z and Y are column vectors of gravity mode and Rossby mode expansion coefficients, respectively; Λ_Z, Λ_Y are diagonal matrices whose elements are the individual eigenfrequencies of the normal modes; N_Z,N_Y are the projections of the nonlinear and forcing terms of the model on the sets of normal modes Z and Y, respectively.

The terms $-i\Lambda_Z Z$ and $-i\Lambda_Y Y$ come from the linear terms of the model equations, which appear in this diagonalized form because the normal modes are eigenfunctions of the linearized equations. We note that because N is a nonlinear operator, it is a function of all the normal mode expansion coefficients Z and Y.

Machenhauer (1977) showed that, for the nonlinear shallow water equations, the linear gravity modes Z contained a low-frequency component as well as the expected high-frequency component. This low-frequency component is essentially due to a balance between the two terms on the right-hand side of Eq. (1). An approximation for the low-frequency balancing component Z_B can be obtained by dropping the time derivative $\dot{Z}$, which gives

$$Z_B = (i\Lambda_Z)^{-1} N_Z(Z,Y). \qquad (3)$$

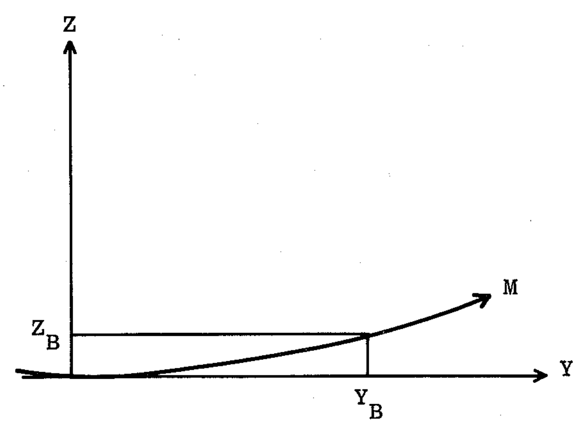

Fig. 1 The slow manifold diagram (after Leith, 1979)

For a f-plane model, Leith (1979) has identified the vector Z_B with the low-frequency ageostrophic flow.

The set of Rossby modes Y plus the low-frequency balancing component Z_B constitutes the slow manifold of Leith (1979). Fig. 1, which is a modified version of Fig. 1 of Leith (1979), illustrates the slow manifold concept. The multi-dimensional phase space of model states, where each dimension is a normal mode amplitude, is schematically represented by a simple two-dimensional diagram. The Rossby mode amplitude Y is the abscissa and the ordinate is the gravity mode amplitude Z. The curved line M represents the locus of low-frequency model states, which Leith (1979) calls the slow manifold. The projection of M on Z gives the balancing component Z_B. Provided the nonlinear terms of the model are not too large, the balancing component Z_B is relatively small compared to Y, so that the slow manifold is not far from the Y axis.

This slow manifold concept has been used to interpret four-dimensional data assimilation by Daley and Puri (1979).

As a matter of fact, the degrees of freedom of the data field expanded in normal modes is always in number more than the number of gravity and/or computational modes which should be filtered out. This means that there exists many possible ways to modify the data field to filter out the gravity and/or computational high frequency oscillations. Based on this argument Daley (1978) developed a variational formulation, and applied it to the shallow water equations. Various weights are used in accordance with the confidence in the original fields. Most existing initialization methods seem to be special cases with particular weights.

5. BASIC STATE

For all normal mode initialization methods discussed above, the basic state of the atmosphere is assumed to be at rest. However, in reality, the atmosphere always has a sheared mean zonal wind.

Some evidence of the unsatisfactory performance of normal mode initialization has been found in regions of high topography. Temperton and Williamson (1979) have given an explanation of these phenomena. Over high topography there are large deviations from the chosen basic state from which the normal modes have been computed. The linear initialization procedure is in effect unaware of the presence of topography, and thus misinterprets these large deviations as gravity waves, which are then removed.

Although this kind of deficiency may be reduced by the non-linear forcing correction in non-linear normal mode initialization, it is quite desirable to use a more realistic basic state in obtaining normal modes. So, in parallel with the assumption of a basic state at rest in operational initialization, the effect of latitudinally varying zonal wind on the normal modes has throughout been the subject of researches.

Dicksinson and Williamson (1972) had tackled this problem numerically in their grid point model, using a very stable barotropic zonal wind profile. No complex eigen-frequencies appear in their calculation. They had found that the mean wind changes the frequencies by both a Doppler shift and by changing the effective Coriolis parameter. Small but noticeable changes in latitudinal structures are observed for low-order Rossby modes. The higher-indexed Rossby modes are drastically affected in latitudinal structure. But for the gravity modes the changes caused by mean zonal wind are within graphical accuracy.

Although a zonal wind exists, these authors discard the latitudinal variation of geopotential. This of course is physically inconsistent. Machenhauer (1977) used the balance equation to determine the latitudinal variation of geopotential from the given non-divergent zonal velocity field. The latter in turn is determined from the observational data. Some weak instability with an e-folding time of about 25 days was observed in the computation for some of the Rossby modes with low zonal wave numbers.

Although Machenhauer's device was rid of physical inconsistency, it, to some extent, breaks off relations with the forecasting equations to use a non-divergent wind-pressure field. This may impair the non-linear normal mode initialization methods.

In our opinion, the best way to take a more realistic basic state into consideration is to appeal to the forecasting equations. The idea is as follows :
To use the non-linear normal mode methods, the forecasting equations are linearized at the beginning. In the linearized equations only the terms of the first order of magnitude are retained and the terms of finite quantities are considered as known. So the linearized equations must be controlled by the zero order equations. For the shallow water equations in spherical coordinates, the zero order equations have the following form :

$$\frac{U^2}{a} \, tg\theta + 2\,\omega \sin\theta \; U + \frac{g}{a}\frac{dH}{d\theta} = 0 \tag{4}$$

in which θ is latitude, ω the angular velocity and a the radius of the Earth.
the Earth.

From this equation we can see that if H is taken as constant, then U must be either zero, or equal to $- 2\,\omega\, a\cos\theta$. The latter case means the atmosphere is in a solid rotation.

Du, Zhou and Di (1979) supposed the following empirical formula for the latitudinal variation of mean zonal wind:

$$U(\theta) = \begin{cases} -U_E \cos^n \dfrac{9\theta}{2}\,, & \theta \le \dfrac{\Pi}{9}\,, \\ U_W \cos^n \left(\dfrac{18}{7}\theta - \dfrac{11}{14}\Pi\right)\,, & \dfrac{\Pi}{9} \le \theta \le \dfrac{\Pi}{2}\,, \end{cases} \tag{5}$$

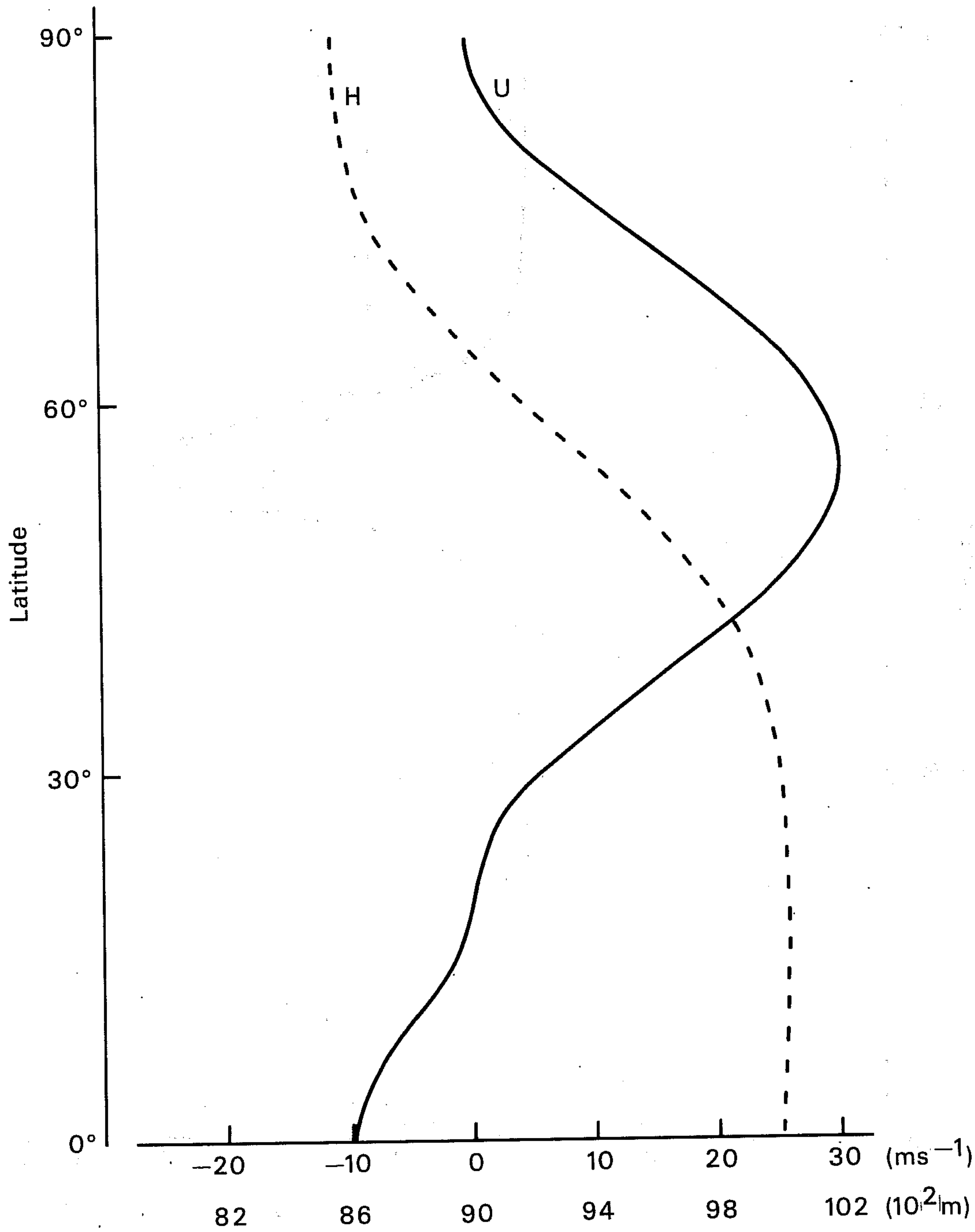

Fig. 2 The mean zonal wind (solid lines in metres per second) and height (dashed lines in hundred metres) profiles for non-jet pattern.

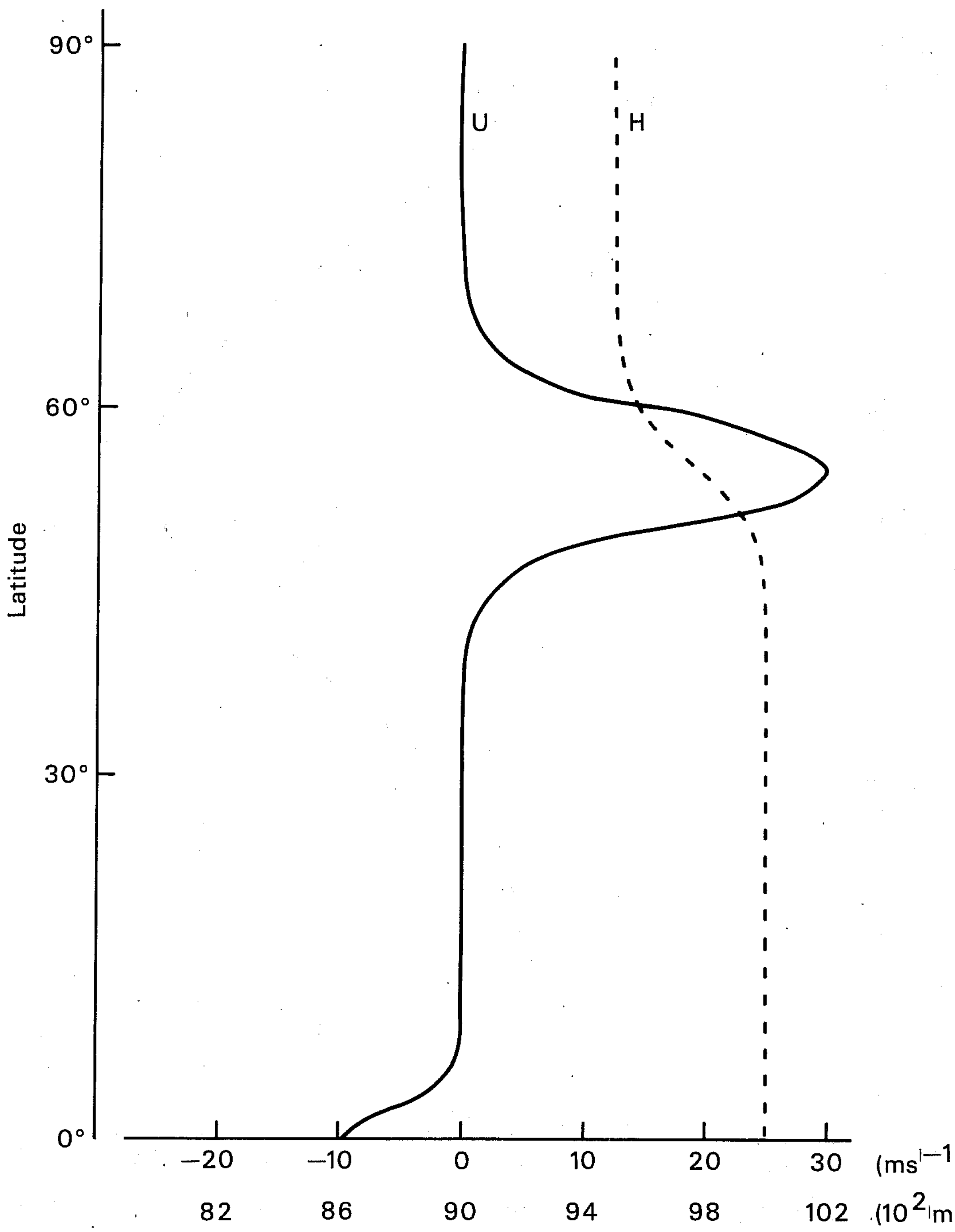

Fig. 3 The mean zonal wind (solid lines in metres per second) and height (dashed lines in hundred metres) profiles for jet pattern.

The latitudinal variation of mean zonal height can be obtained by integrating Eq. (4). The results are given in Fig. 2 and Fig. 3. Fig. 2 represents a non-jet pattern in which $n = 2$, and Fig. 3 is typical for jet pattern in which $n = 22$. These mean zonal winds (solid lines) and heights (dashed lines) profiles are used in calculating normal modes. One can see that for these wind profiles barotropic instability should occur, see Kuo (1949). But since a finite-difference method is used in the north-south direction, the barotropic growth may be obscured by the truncation error. The results of ordinary second-order finite difference schemes are compared with that of the fourth-order scheme. The result of 4th order differencing is significantly better, and does reveal more barotropic growth. The e-folding times of the barotropic instability, in general, are about 10 - 20 days, - a little shorter than Machenhauer's.

The effects of latitudinal shear on equatorial waves have been studied by Boyd, (1978), Du and Zhou (1980) with different approximate methods. Some kind of eastward propagating Rossby waves may be induced by easterly wind shear. Their amplitude and frequency tend to zero as the shear tends to zero. The sensitivity of the initialization to the assumption of a resting basic state for the calculation of normal modes is also presently being investigated by Williamson and others.

6. NUMERICAL SCHEMES

From the above analyses, one can see that the normal mode analysis can also provide information on the assessment and validity of various numerical schemes. It is helpful to find means of improving the stability of a computational scheme, and increasing the accuracy in order not to mask the physical essence. In this direction we can still list the following examples.

Applying Arakawa's (1972) staggered finite-difference formulation (C-grid) to the ECMWF grid point forecast model with H defined at points (i,j), u at $(i + \frac{1}{2}, j)$ and v at $(i, j + \frac{1}{2})$, Temperton (1977) had computed the normal modes of the shallow-water equation. The basic state of the atmosphere is assumed to be at rest, and an ordinary second-order difference scheme is used. The numerical normal modes on the staggered 10° grid resemble those of the non-staggered 5° grid. The computational Rossby modes, which appear on the non-staggered grid, do not appear on the staggered grid.

Compared with the second-order finite difference scheme, the results obtained by the fourth-order finite difference scheme in non-staggered grid are nearer to Temperton's (1977) results. The three kinds of results for frequencies of Rossby modes for $H = 10$ km, $k = 1$, are presented in Table 1.

Table 1. The frequencies (s^{-1}) of Rossby modes in some different numerical schemes. L is an index for the frequencies.

L	2nd-order staggered	2nd-order non-staggered	4th-order non-staggered
0	$6.11 . 10^{-5}$	$6.07 . 10^{-5}$	$6.3188 . 10^{-5}$
1	$1.44 . 10^{-5}$	$1.39 . 10^{-5}$	$1.3918 . 10^{-5}$
2	$8.64 . 10^{-6}$	$8.08 . 10^{-6}$	$8.6623 . 10^{-6}$
3	$5.72 . 10^{-6}$	$5.12 . 10^{-6}$	$5.5311 . 10^{-6}$
4	$3.98 . 10^{-6}$	$3.32 . 10^{-6}$	$3.9347 . 10^{-6}$
5	$2.87 . 10^{-6}$	$2.13 . 10^{-6}$	$2.6239 . 10^{-6}$
6	$2.14 . 10^{-6}$	$1.27 . 10^{-6}$	$1.8867 . 10^{-6}$
7	$1.63 . 10^{-6}$	$5.96 . 10^{-7}$	$1.1350 . 10^{-6}$
8	$1.27 . 10^{-6}$	$-2.07 . 10^{-19}$	$8.5484 . 10^{-7}$
9	$1.01 . 10^{-6}$	$-5.96 . 10^{-7}$	$3.1890 . 10^{-8}$
10	$8.10 . 10^{-7}$	$-1.27 . 10^{-6}$	$-4.7556 . 10^{-7}$
11	$6.62 . 10^{-7}$	$-2.13 . 10^{-6}$	$-1.3028 . 10^{-6}$
12	$5.52 . 10^{-7}$	$-3.32 . 10^{-6}$	$-2.2466 . 10^{-6}$
13	$4.70 . 10^{-7}$	$-5.12 . 10^{-6}$	$-4.0780 . 10^{-6}$
14	$4.11 . 10^{-7}$	$-8.08 . 10^{-6}$	$-7.0341 . 10^{-6}$
15	$3.75 . 10^{-7}$	$-1.39 . 10^{-5}$	$-1.4018 . 10^{-5}$
16	$3.13 . 10^{-7}$	$-6.07 . 10^{-5}$	$-7.9917 . 10^{-5}$

Similar results for frequencies of eastward gravity modes are given in Table 2.

Table 2. The frequencies (s^{-1}) of eastward gravity modes in some different numerical schemes. L is an index for the frequencies.

L	2nd-order staggered	2nd-order non-staggered	4th-order non-staggered
0	$-5.44 \cdot 10^{-5}$	$-5.35 \cdot 10^{-5}$	$-5.4134 \cdot 10^{-5}$
1	$-1.31 \cdot 10^{-4}$	$-1.28 \cdot 10^{-4}$	$-1.3263 \cdot 10^{-4}$
2	$-1.87 \cdot 10^{-4}$	$-1.81 \cdot 10^{-4}$	$-1.8888 \cdot 10^{-4}$
3	$-2.35 \cdot 10^{-4}$	$-2.22 \cdot 10^{-4}$	$-2.3768 \cdot 10^{-4}$
4	$-2.79 \cdot 10^{-4}$	$-2.55 \cdot 10^{-4}$	$-2.8217 \cdot 10^{-4}$
5	$-3.22 \cdot 10^{-4}$	$-2.78 \cdot 10^{-4}$	$-3.2321 \cdot 10^{-4}$
6	$-3.63 \cdot 10^{-4}$	$-2.93 \cdot 10^{-4}$	$-3.5866 \cdot 10^{-4}$
7	$-4.01 \cdot 10^{-4}$	$-3.11 \cdot 10^{-4}$	$-3.8082 \cdot 10^{-4}$
8	$-4.36 \cdot 10^{-4}$	$-3.86 \cdot 10^{-4}$	$-3.9659 \cdot 10^{-4}$
9	$-4.69 \cdot 10^{-4}$	$-3.86 \cdot 10^{-4}$	$-4.4134 \cdot 10^{-4}$
10	$-4.98 \cdot 10^{-4}$	$-3.09 \cdot 10^{-4}$	$-4.4117 \cdot 10^{-4}$
11	$-5.23 \cdot 10^{-4}$	$-2.85 \cdot 10^{-4}$	$-3.9006 \cdot 10^{-4}$
12	$-5.44 \cdot 10^{-4}$	$-2.60 \cdot 10^{-4}$	$-3.5574 \cdot 10^{-4}$
13	$-5.61 \cdot 10^{-4}$	$-2.30 \cdot 10^{-4}$	$-3.1235 \cdot 10^{-4}$
14	$-5.72 \cdot 10^{-4}$	$-1.97 \cdot 10^{-4}$	$-2.5736 \cdot 10^{-4}$
15	$-5.94 \cdot 10^{-4}$	$-1.65 \cdot 10^{-4}$	$-1.9942 \cdot 10^{-4}$
16	$-5.94 \cdot 10^{-4}$	$-1.31 \cdot 10^{-4}$	$-1.5162 \cdot 10^{-4}$

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