# Quantifying EPS forecast skill and multiscale predictabilty of hydrometeorological fields

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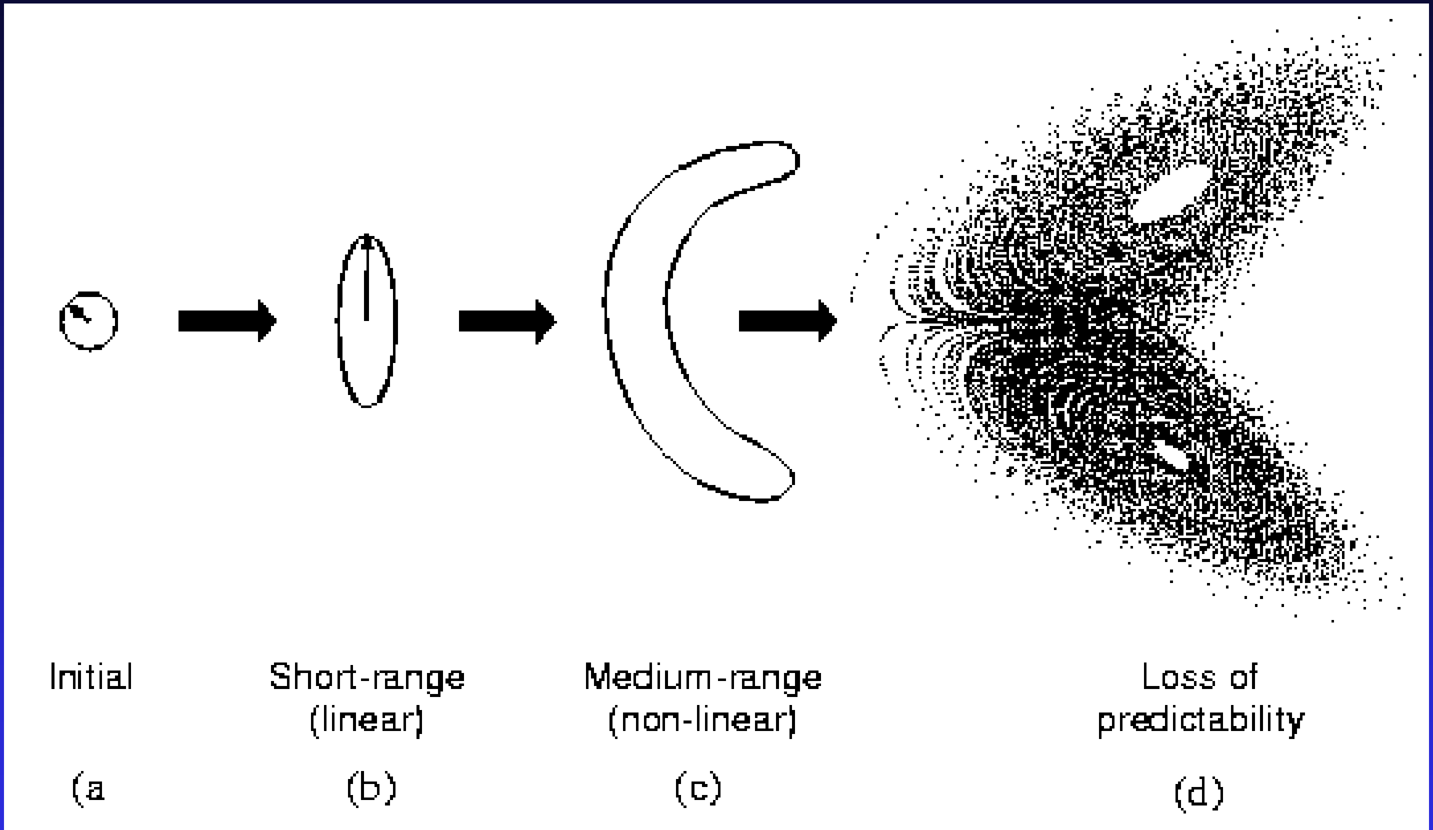
# EPS and lessons from chaos?

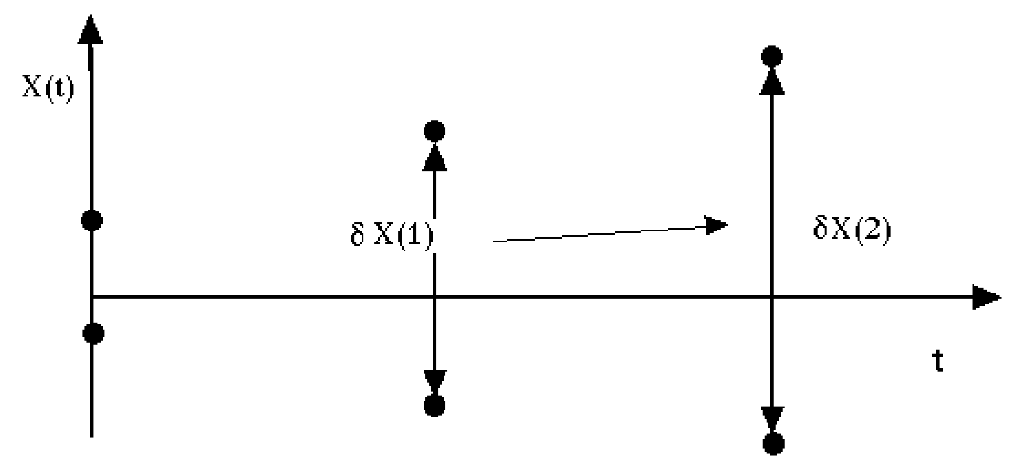



Scheme of the evolution of the empirical pdf in EPS
Phase space (according to Palmer 1999): initial ensemble (a),
to (b) linear growth phase, to (c) nonlinear growth phase, to (d) loss of predictability.

# Exponential error growth: $|\delta X(t)| \approx e^{\mu t} |\delta X(0)|$

- Heuristics:

$$X(t) = G(X(t-1)) \quad X \in R$$




$$|\delta X(t)| \approx |D_{X(t-1)} G| \, |\delta X(t-1)|$$

$$Log\left[|\delta X(t)| / |\delta X(0)|\right] \approx \sum_{t'=0,t-1} Log\left(|D_{X(t')} G|\right) \approx t \langle |D_x G| \rangle = t\,\mu$$

- Multiplicative Ergodic Theorem (Oseledec, 1968):
  - if and only if: $\mu = \int Log^{+}(\|A(x)\|) d\rho(x) < \infty$

A being the generator of the flow

•limited extension for infinite dimensions.

# EPS and Fokker-Planck equation ?

On a **finite** d-dimensional phase space $\{X_1, X_2, ..., X_d\}$

$$\dot{\underline{X}}(t) = \frac{d}{dt}\underline{X} = \underline{F}(\underline{X}, t) \quad \Rightarrow$$

$$\frac{\partial}{\partial t}\rho(\underline{X}, t) + \sum_{i=1}^{d} \frac{\partial}{\partial X_i}\left[\dot{X}_i(t)\rho(\underline{X}, t)\right] = 0$$

for the pdf $\rho(\underline{X}, t)$ w.r.t. $dX_1 dX_2 ... dX_d$

But noisy perturbations due to subgrid scales $\frac{d}{dt}\underline{X} = \underline{F}(\underline{X}, t) + \underline{f}(t)$

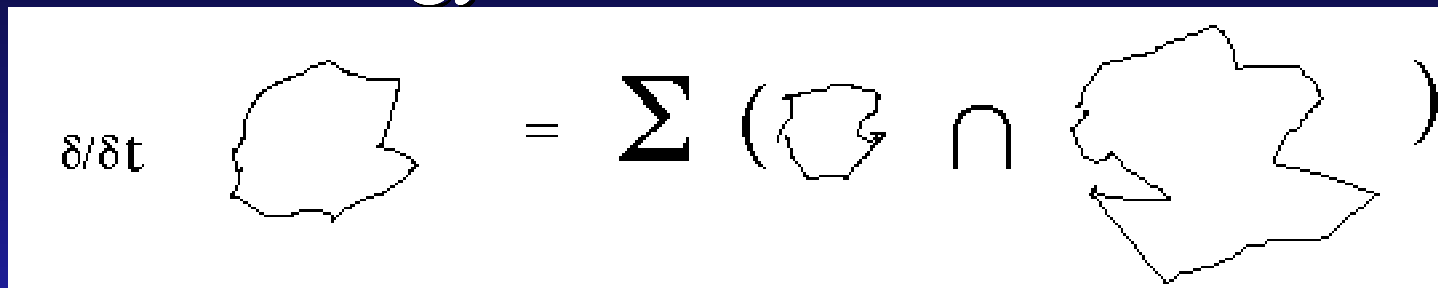
If gaussian:

$$\langle f_i(t) f_j(t') \rangle = \varepsilon \delta_{i,j} \delta(t - t') \Rightarrow \frac{\partial}{\partial t}\rho(\underline{X}, t) + \sum_{i=1}^{d} \frac{\partial}{\partial X_i}\left[\dot{X}_i(t)\rho(\underline{X}, t)\right] - \varepsilon \Delta_X \rho(\underline{X}, t) = 0$$

If non-gaussian, e.g. Lévy type: $-\Delta_X \to (-\Delta_X)^\alpha$ (+other fractional operators)

# Power-law decorelation: $\ell \approx t^{3/2}$

- Phenomenology:




by nonlinear interactions, uncertainties grow up to larger scales

- Heuristics :

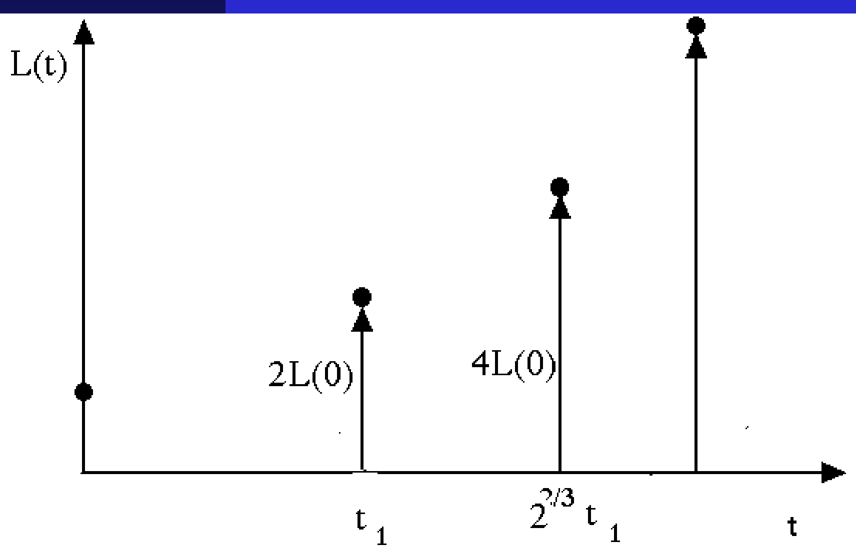
$t \approx \ell / \delta u(\ell)$

eddy turn-over time

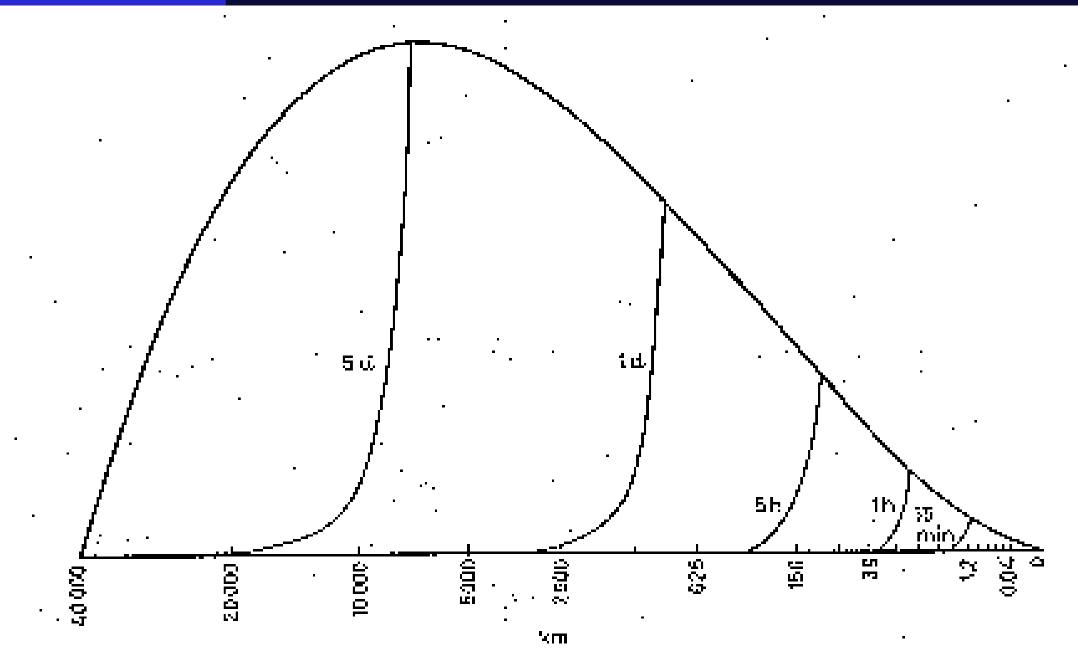
$\ell \approx t^{3/2}$

$\varepsilon \approx \delta u(\ell)^3 / \ell$

scale invariance of the energy flux (K41)



# Spectral analysisand closures




$$\lambda_c \approx t^{3/2}$$

Lorenz (1969,
Leith and Kraichnan(1972),
Metais and Lesieur (1986)

Flux from correlated energy

$$e^c(\underline{x},t) = \tfrac{1}{2}\underline{u}^2(\underline{x},t).\underline{u}^1(\underline{x},t)$$

to decorrelated energy

$$e^\Delta(\underline{x},t) = \tfrac{1}{2}\left(\underline{u}^2(\underline{x},t) - \underline{u}^1(\underline{x},t)\right)^2$$

Similar results with: $\lambda_c^{2/3} = \bar{\varepsilon}^{1/3} t^{3/2};\ \bar{\varepsilon} = 10^{-3}\,\mathrm{m}^2\mathrm{s}^{-3}, \eta \approx 10^{-3}\,\mathrm{m};$

# Questioning the Lorenz-Leith-Kraichnan approach

- 2 fundamental comments (Lilly, 1985):
  - appraisal of Lorenz 's choice of 3D energy spectrum up to synoptic scales, initialy considered as rather paradoxical:

  what is the dimension of atmospheric dynamics?

  - strong criticism of the homogenity and quasi-normal hypotheses of closures:

  what are the statistics?

# Richardson Diffusion law

Richardson (1926) found a scaling turbulent viscosity up to planetary scales:

- $\nu(L) \approx \varepsilon^{1/3}\ L^{4/3}$

associated to an anomalous diffusion:

- $< r(t)^2 > \approx \varepsilon\ t^3$

the first quantitative law in turbulence..

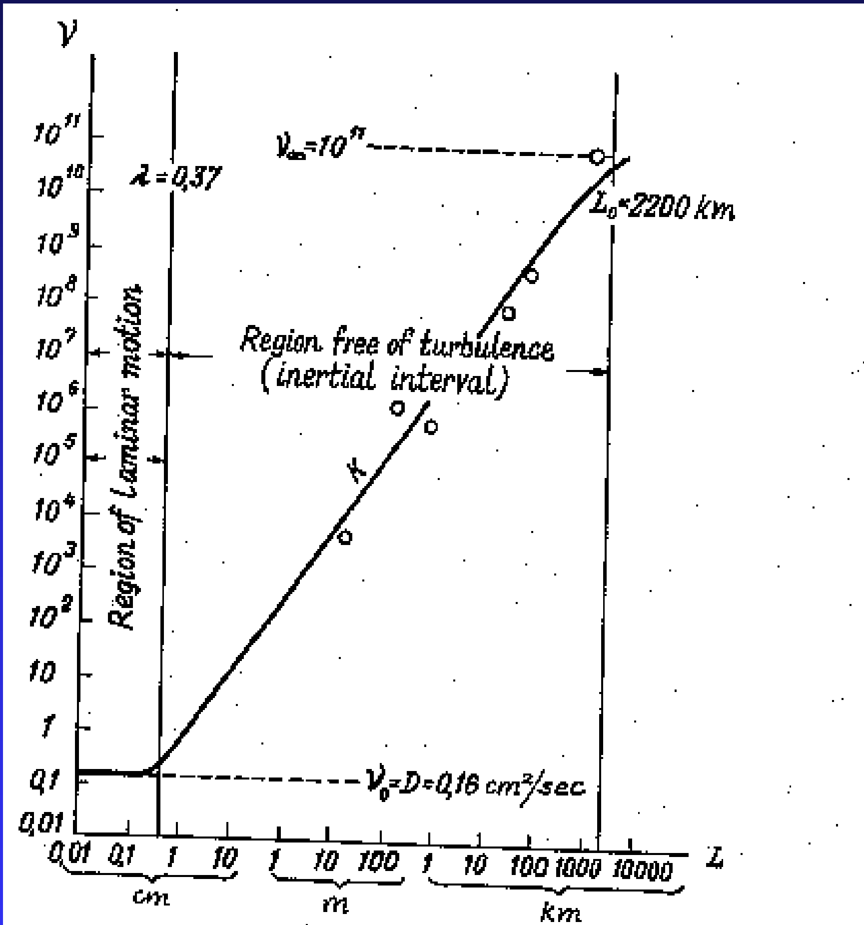
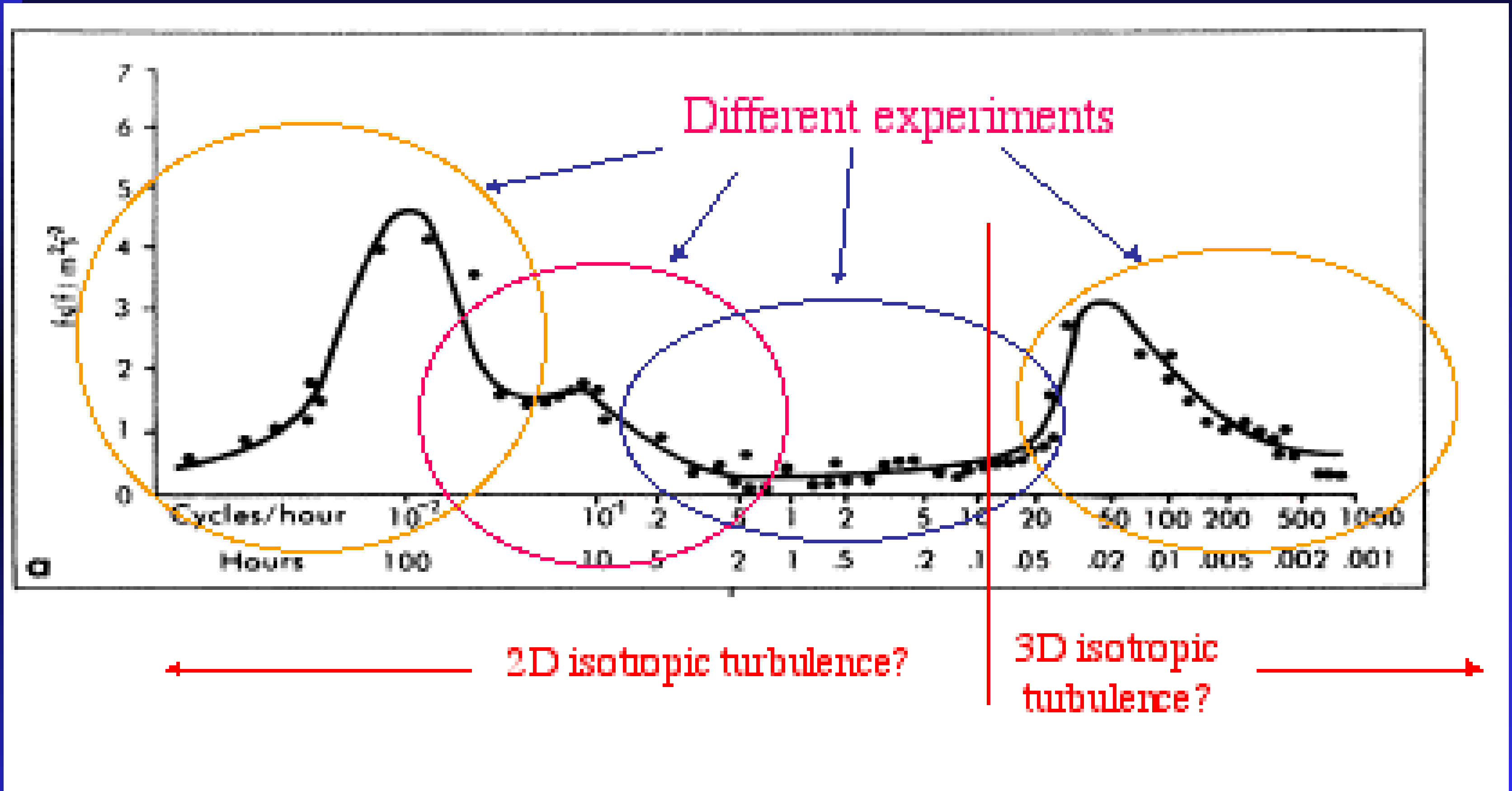


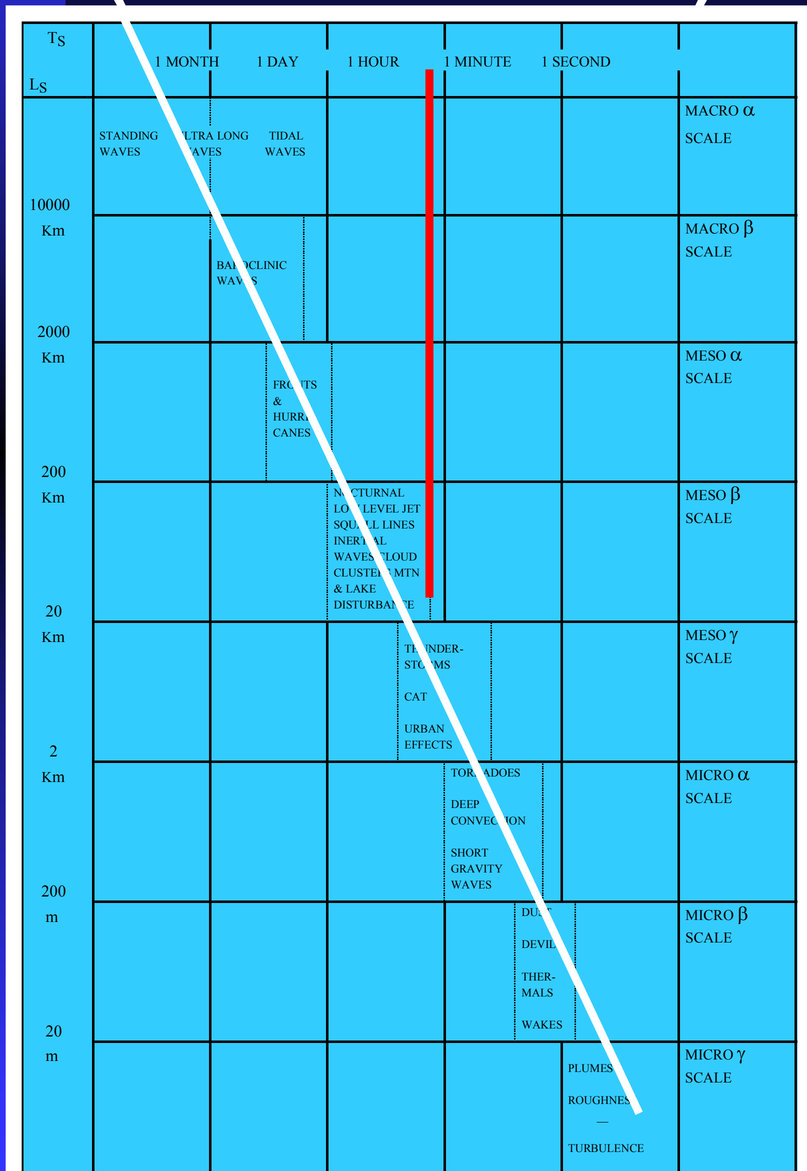

Fig. 1 The vertical diffusion coefficient $\nu(L)$ as a function of the turbulence scale $L$. Empirical points after Richardson.[19]

# Van der Hoven wind spectrum (1957)




Richardson cascade is split into macro, meso, micro oscillations...

# Space-Time ("Stommel") diagram




Schematic diagram showing a typical phenomenologist's view of meteorology, reproduced from Atkinson (1981)who adapted it from Orlanski (1975) .

The added straight line corresponds to :

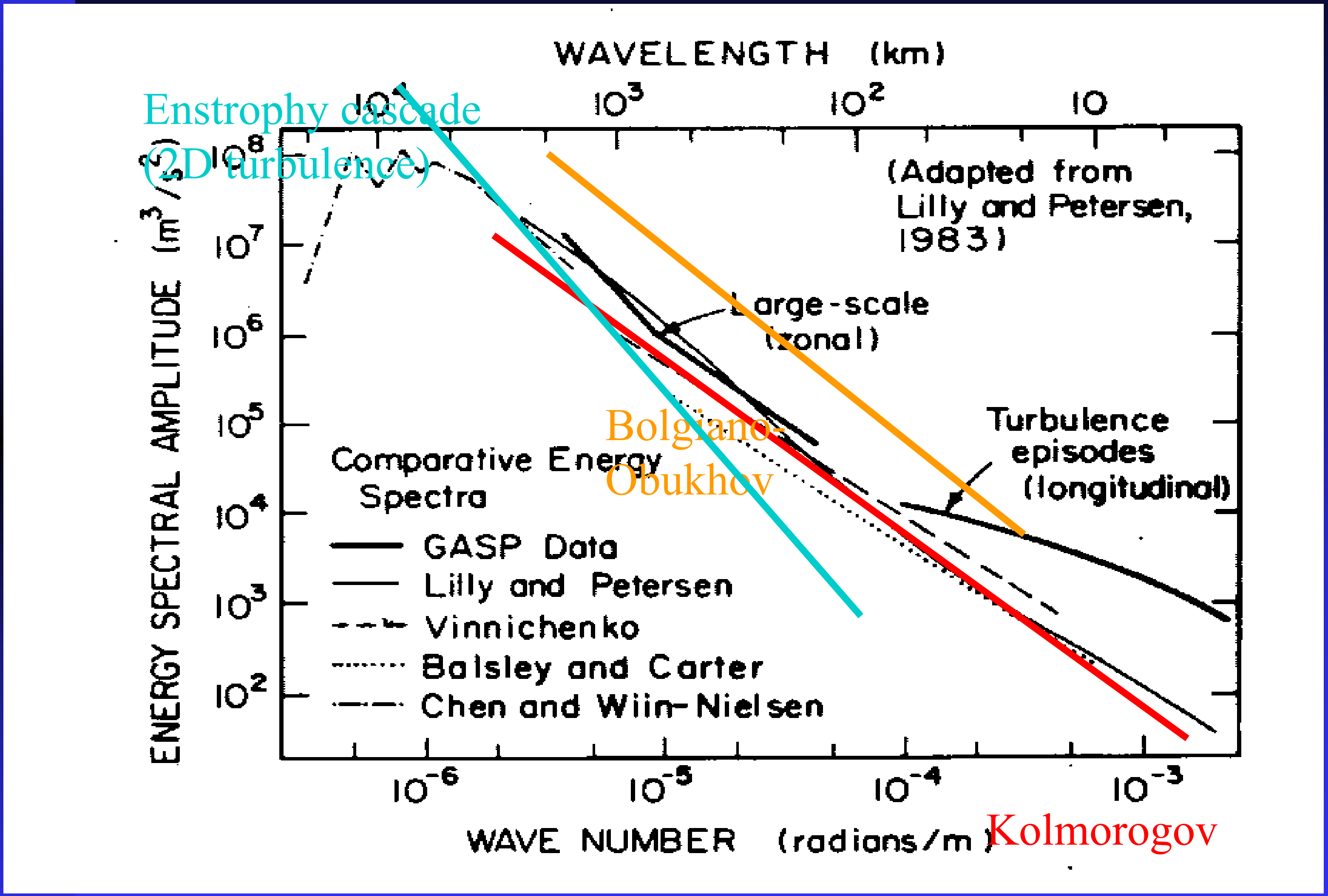
$$T \approx \varepsilon^{-1/3} L^{2/3}$$

in agreement with Richardson (1926) and Kolmogorov (1941), **i.e. 3D-like behavior**

Schertzer et al., Fractals, 1997

The vertical line corresponds to a 2D-like large scale behavior

# Wind: the GASP Experiment



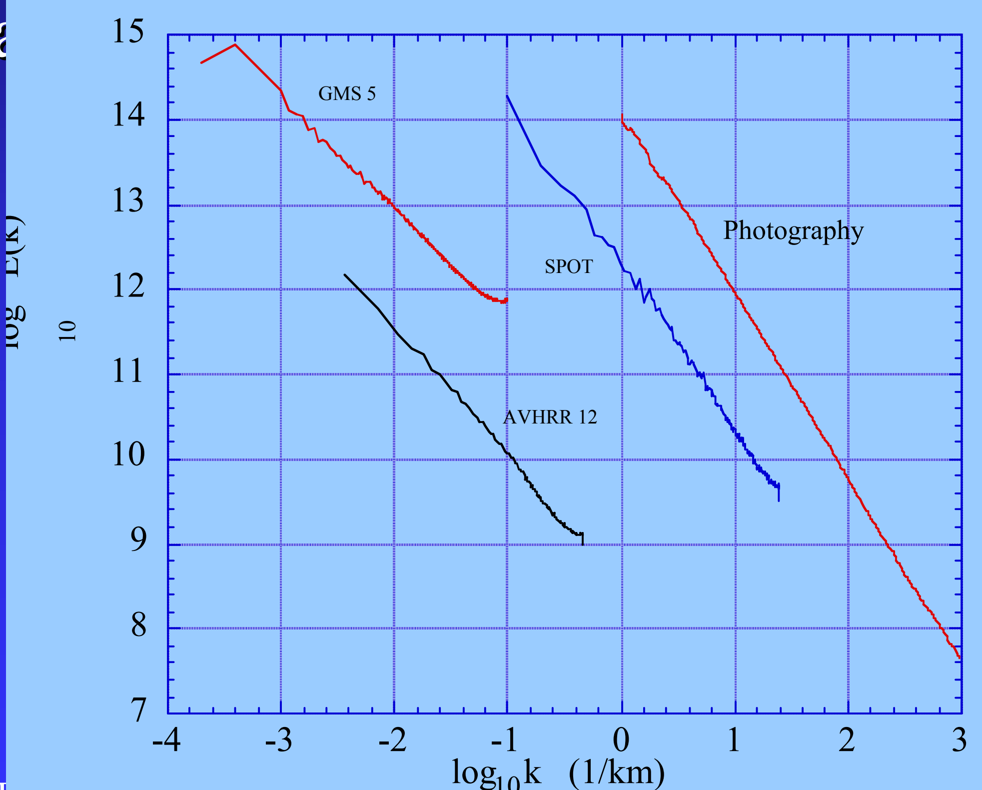

Adapted from Nastrom and Gage 1983

# What is the outer scale of atmospheric turbulence?

$L_{eff} >$ 5 000 km !!

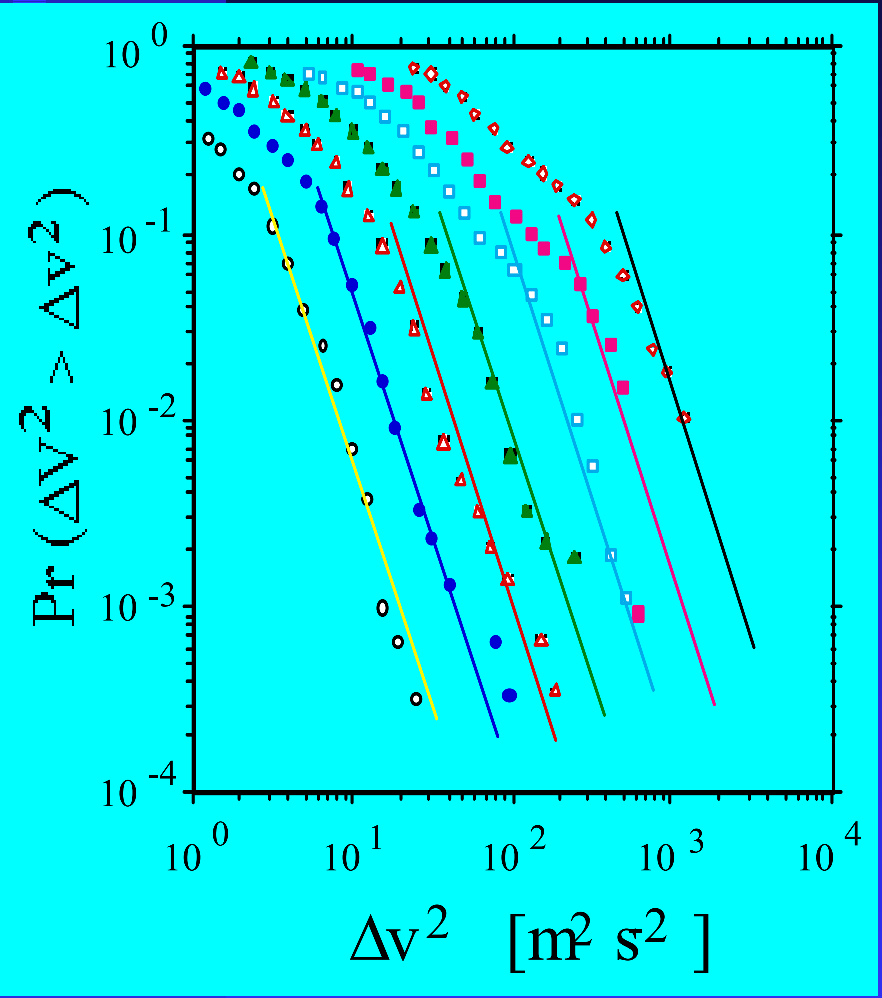
Spectra of hundreds of satellite images spanning the scale range 1-5000 km, and 38 clouds spectra (1m-1km) from ground camera

(Sachs et al, 2001)




A multifractal analysis will be more informative (see below).

# Landes experiment (Météo-France)




80 balloon vertical soundings: the probability distribution of the vertical of shears the horizontal wind, is scaling for Δz = 50,100,200, 400,800, 1600, 3200 m. (*):

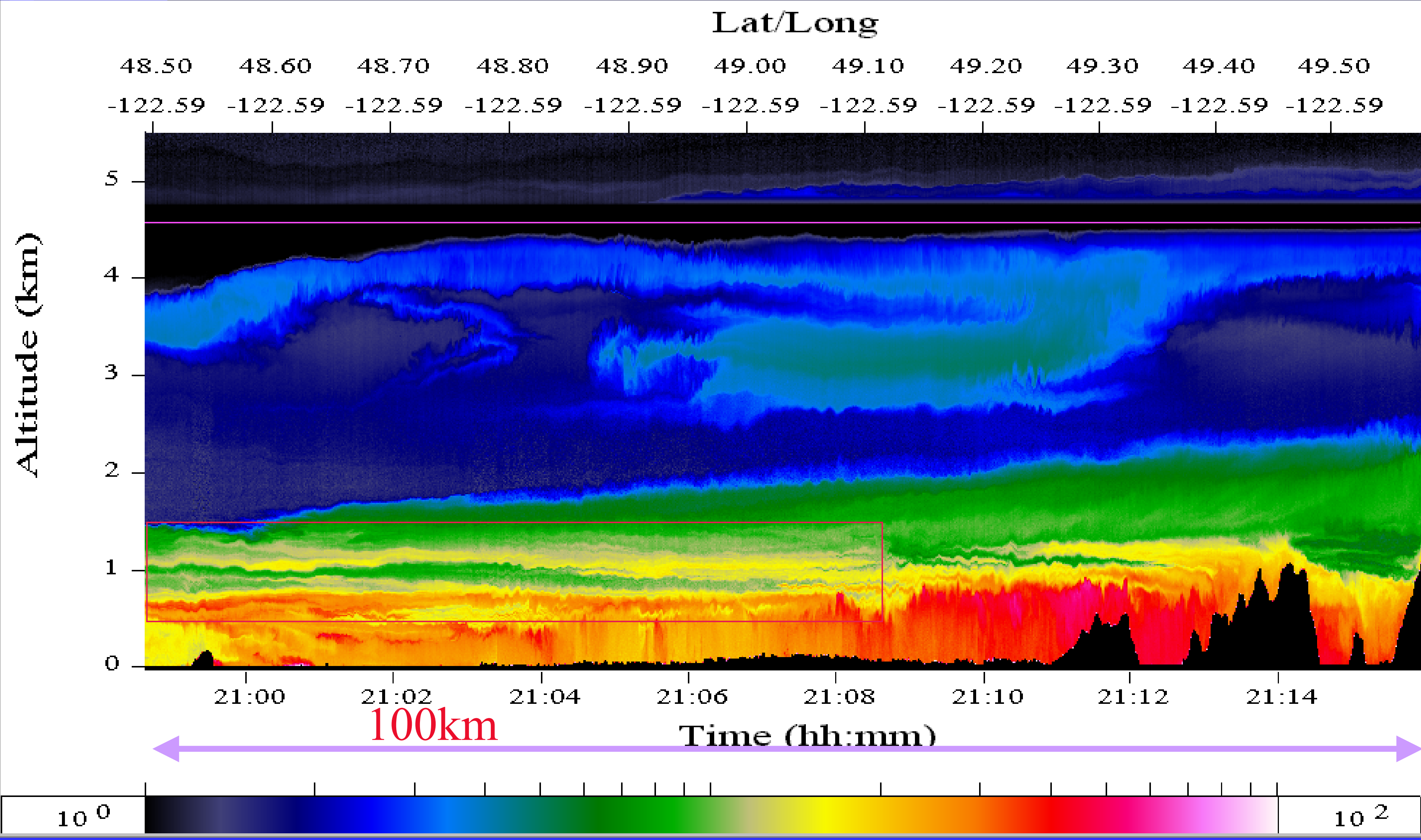
$$\Delta v_z(\Delta z) \sim \Delta z^{3/5}$$

power law tails: $q_D \approx 5$ !

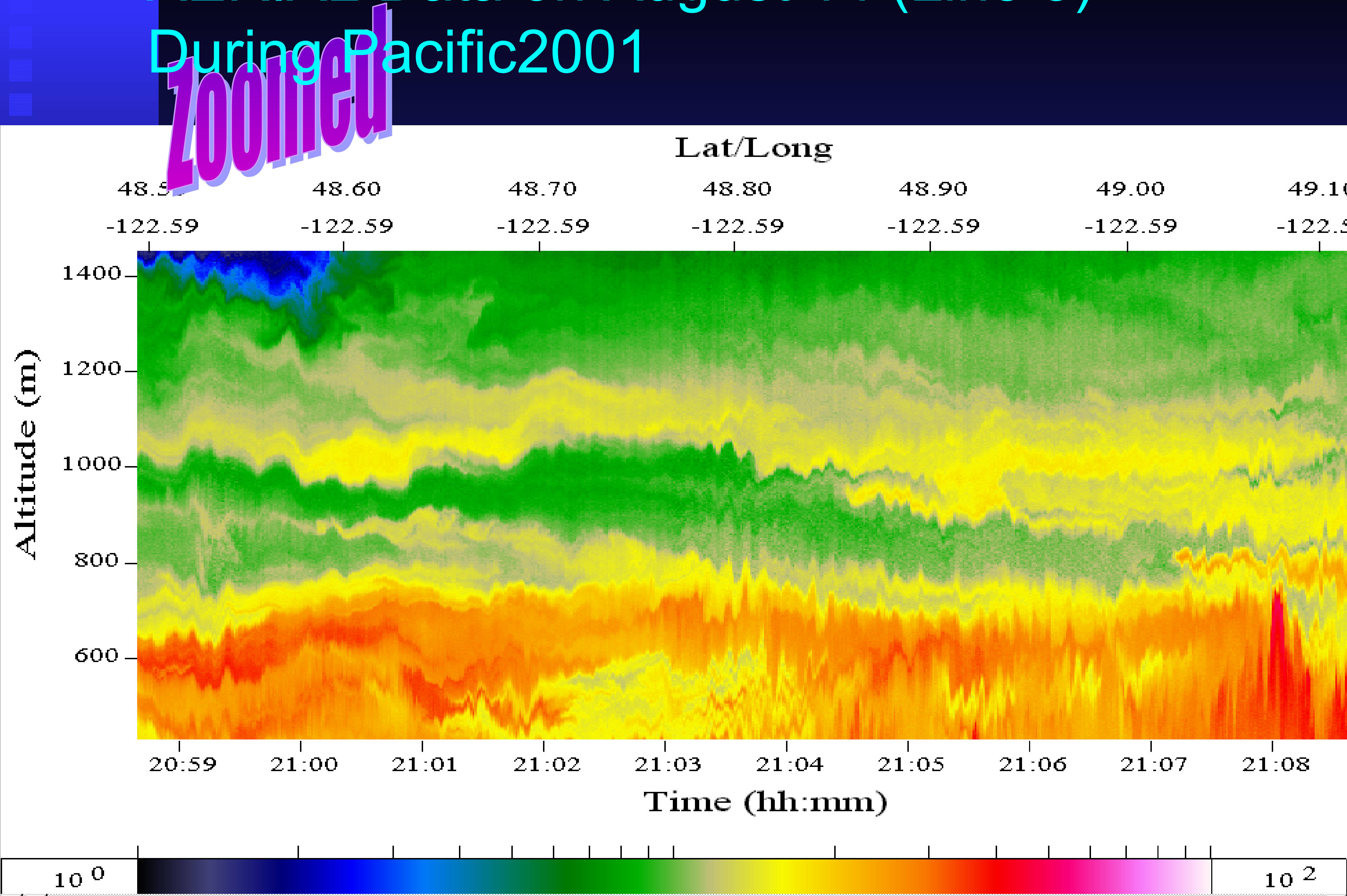
* see also Endlich et al., JAS, 1969, Adelfang, JAS, 1971

Schertzer and Lovejoy,TSF4, 1984

# AERIAL Data on August 14 (Line 5 N-S) During Pacific2001

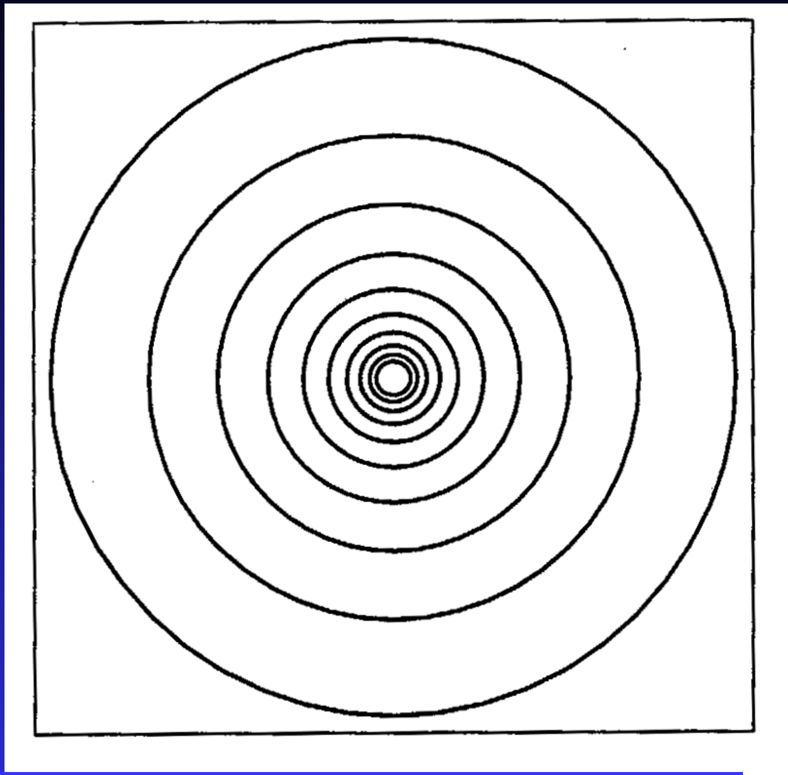


# AERIAL Data on August 14 (Line 5) During Pacific2001



# The physical scale function predicted by the 23/9D model of atmospheric dynamics

Isotropic function



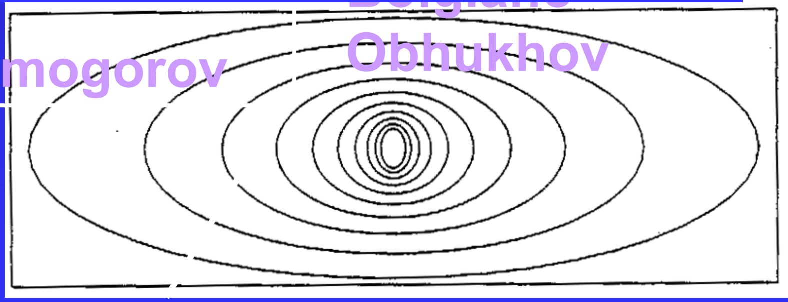
$H_z=1$

- stratification
- flattening of structures

at larger scales

Anisotropic physical scale function

$$\|\Delta r\| = l_s\left(\left(\frac{\Delta y}{l_s}\right)^2+\left(\frac{\Delta x}{l_s}\right)^2+\left(\frac{\Delta z}{l_s}\right)^{2/H_z}\right)^{1/2}$$

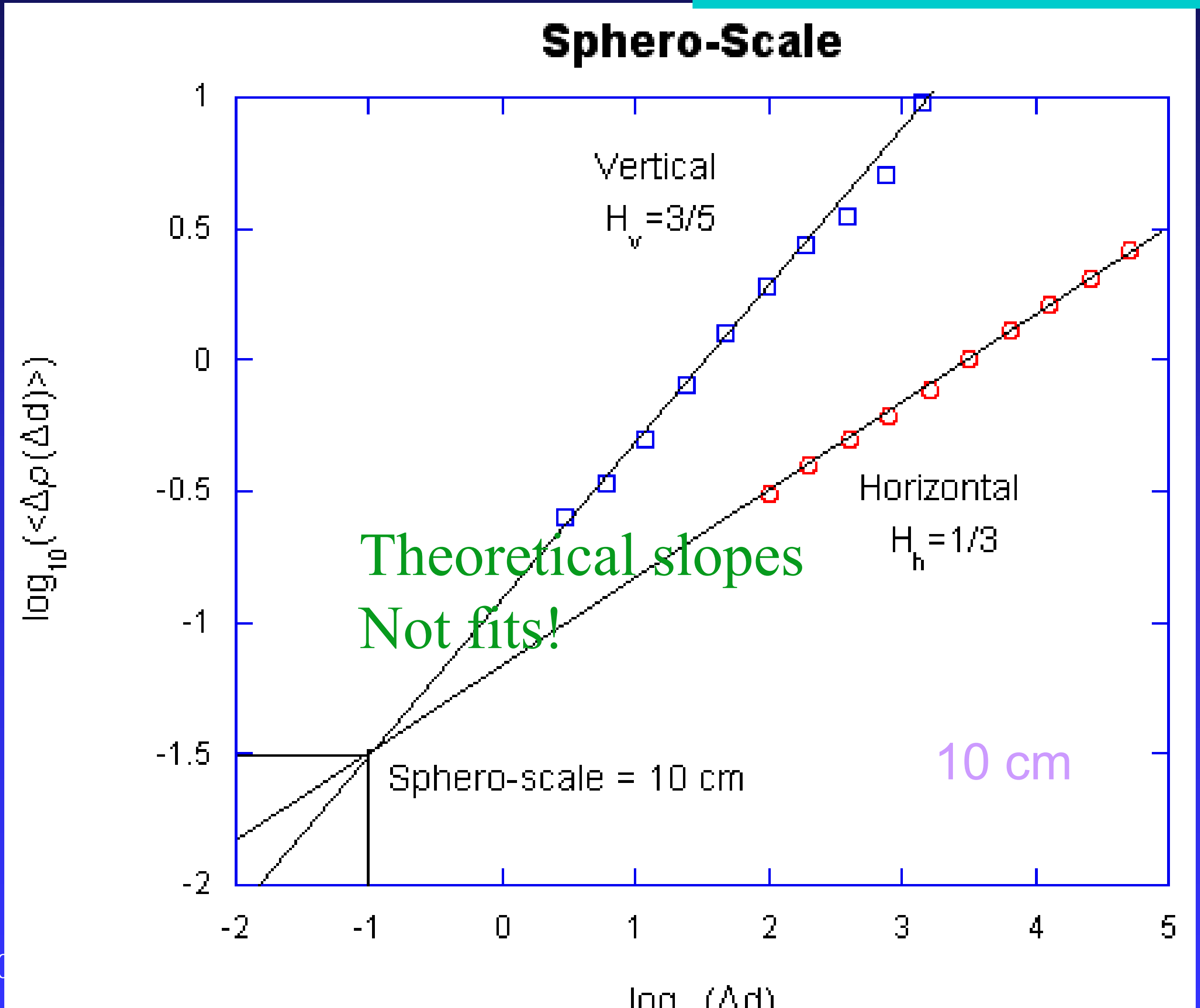



$H_z=5/9$

# Direct Confirmation of physical scale from lidar passive scalars

**Generalized Corrsin-Obhukov law**

$$\Delta\rho(\Delta r) = \chi^{1/2}\varepsilon^{-1/6}\|\Delta r\|^{1/3}$$

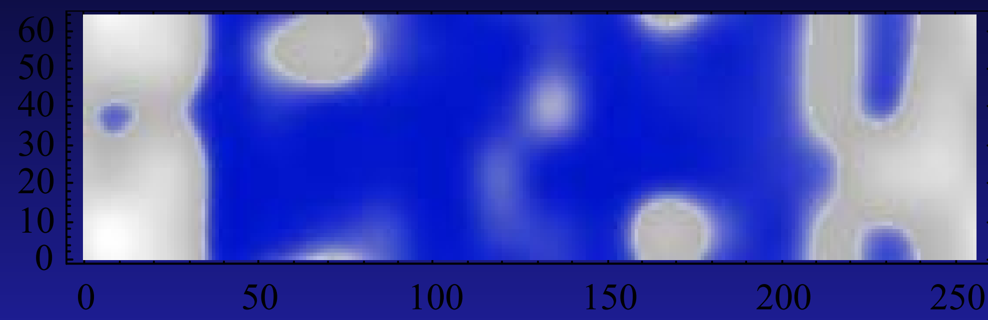



Lilley, L+S, Strawbridge 2003

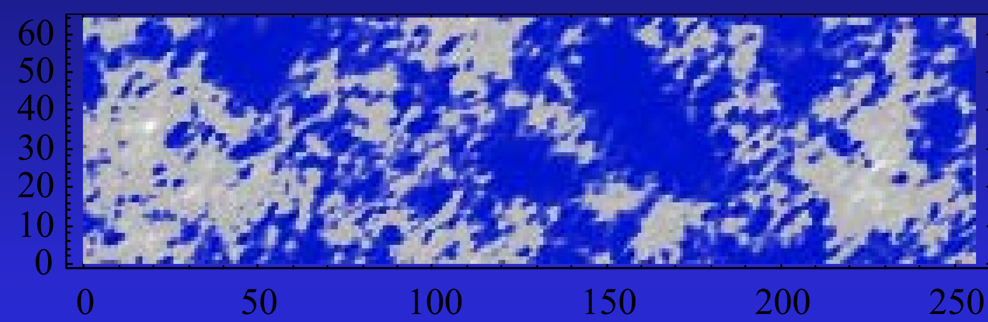
8-10

# Cascade model vs. cascade of models

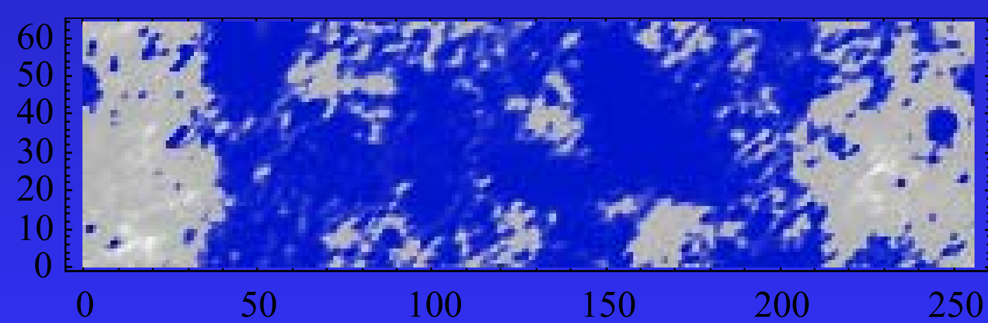
*Contributions form* $l_1$ *to* $l_2$




*Contributions from* $l_2$ *to* $l_3$




One parameter multiplicative group => Gobal result: *contributions from* $l_1$ *to* $l_3$



# Self-similar multifractal cloud

QuickTime™ and a
Animation decompressor
are needed to see this picture.

Downscaling (blow up) factor 1000; $\alpha$=1.8, $C_1$=0.05, H=1/3

# Downscaling of stratified clouds

If $l_s$=1m, then at start: 8km wide, end of zoom: 2.5m wide (factor 32000)

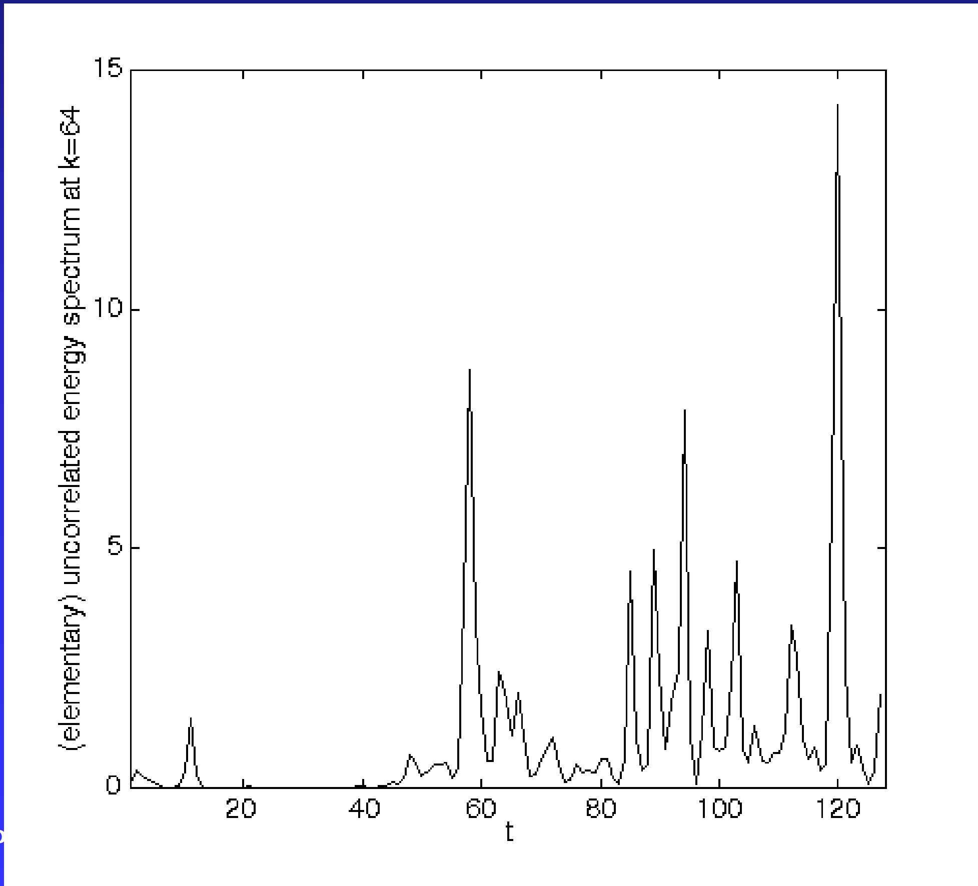
QuickTime™ and a
Animation decompressor
are needed to see this picture.

□=1.8, C1=0.05, H=0.33, d=7/9, c=-2/9, e=0., f=0, ls=2^-5 to (about 2^10 at the 108th image (factor1.1 enlargements). There are 4X4 subpixels, 2^7X2^8.

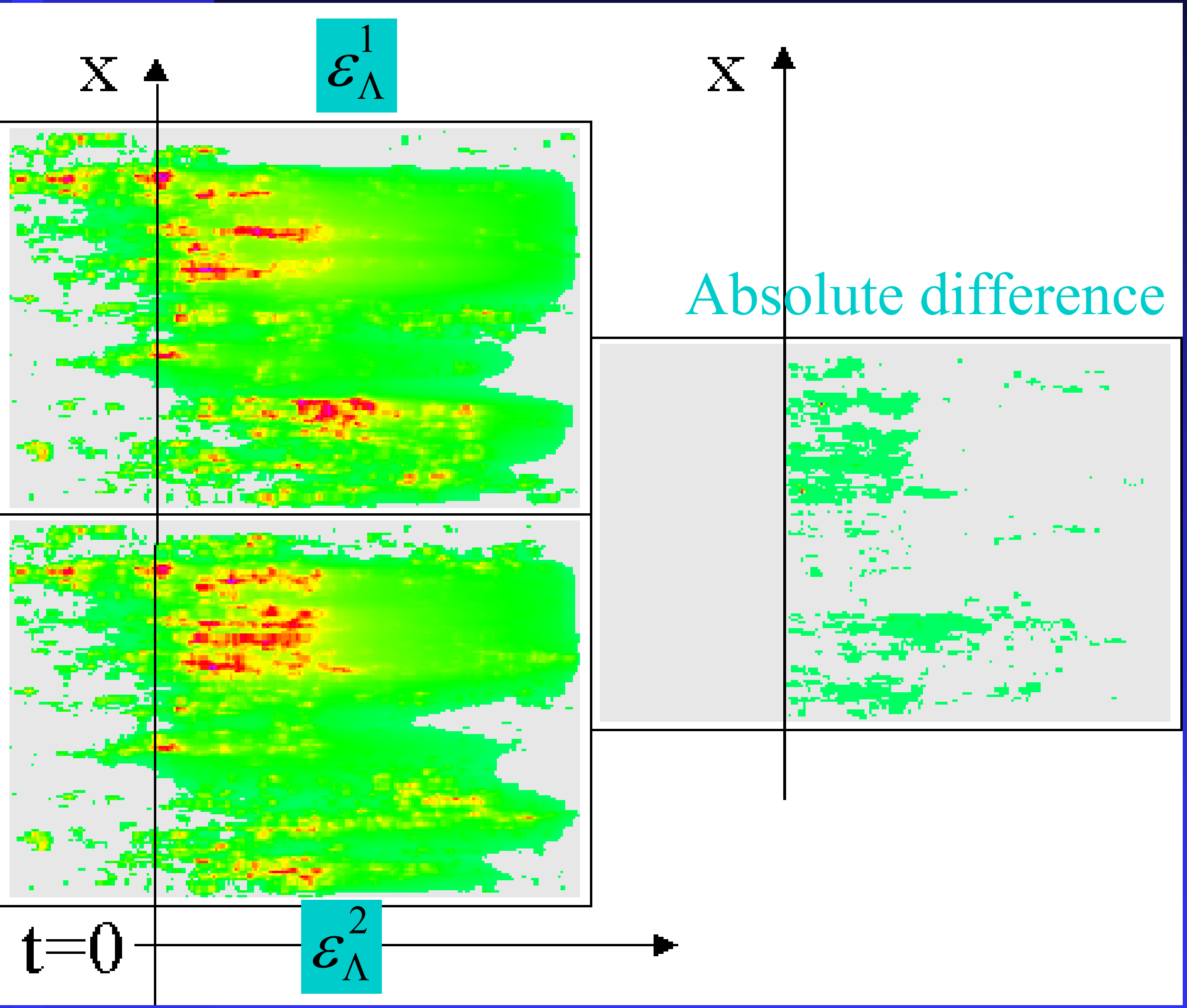
# Multifractal rain simulation

QuickTime™ and a
Animation decompressor
are needed to see this picture.

Fondamental role of intermittency:
Intermittent puffs of decorrelation



# Multifractal predictability limits




Generation of intense Differences, due to Growing independence between the two fields at small scales..

# Multifractality of the joint field $\varepsilon_\Lambda^1 \varepsilon_\Lambda^2$

$$< {\varepsilon_\Lambda}^q > \approx \Lambda^{K(q)} =>$$

$$C^{(q)}(\varepsilon_\Lambda^1, \varepsilon_\Lambda^2) = \left\langle \left(\varepsilon_\Lambda^1 \varepsilon_\Lambda^2\right)^q \right\rangle / \left\langle {\varepsilon_\Lambda^1}^q \right\rangle \left\langle \left(\varepsilon_\Lambda^2\right)^q \right\rangle$$

$$\propto \lambda(t)^{K(q,2)}; K(q,2) = K(2q) - 2K(q)$$

Hint:
the 2 fields remain identical on a decreasing scale ratio:
whereas indepedent for smaller scales.

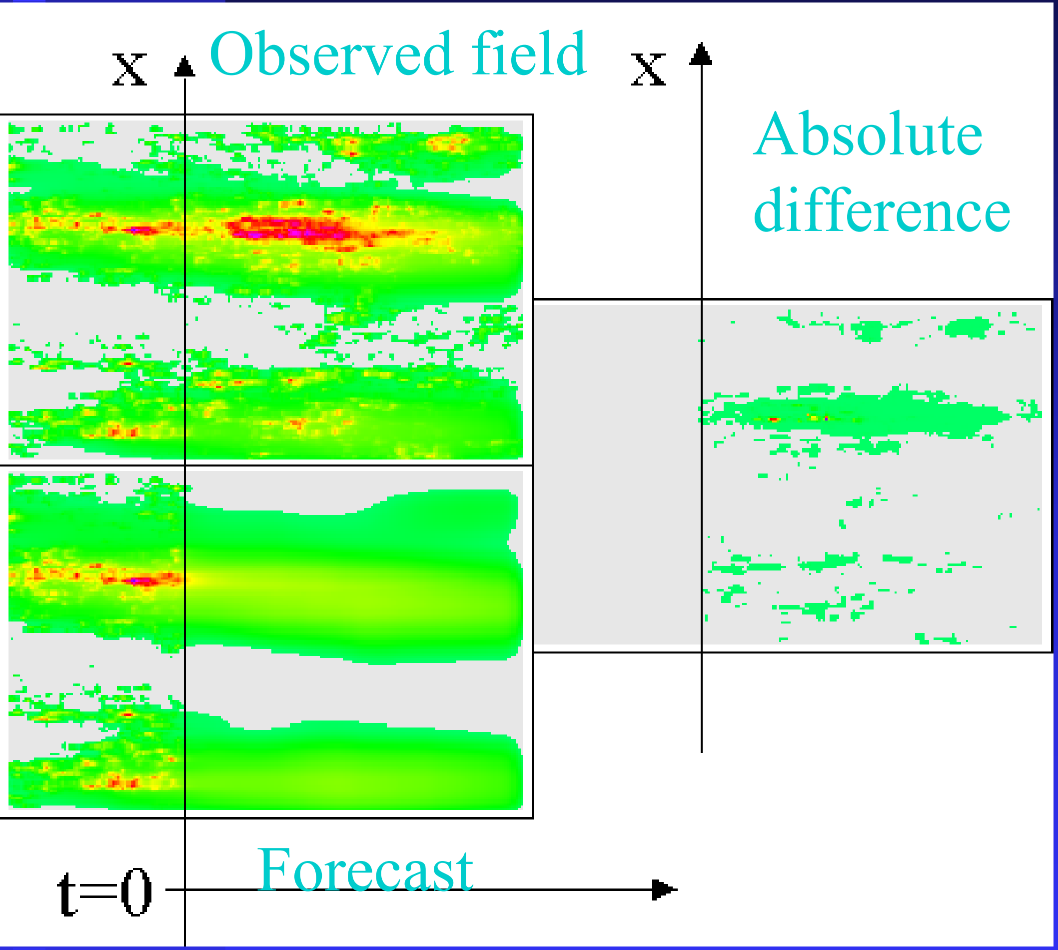
$$\lambda(t) \approx Min\left[\Lambda, (T/t)^{\frac{1}{1-H_t}}\right] \; (t > 0)$$

($1 - H_t$ = dynamical exponent
T = outer time scale)

$$\Pr({\varepsilon_\Lambda}^q > \Lambda^\gamma) \approx \Lambda^{c(\gamma)} => \Pr\left(\left(\varepsilon_\Lambda^1 \varepsilon_\Lambda^2\right) > \lambda(t)^\gamma \left\langle \varepsilon_\Lambda^1 \right\rangle \left\langle \varepsilon_\Lambda^2 \right\rangle\right) \approx \Lambda^{c(\gamma,2)}$$

$c(\gamma), K(q)$
$c(\gamma,2), K(q,2)$
are (merely) dual for the Legendre transform !

# Multifractal forecast and deterministic sub-grid modeling




Future component of the generator deterministically prescribed to preserve the mean field:rapid loss of small structures!

This confirms and explain why Stochastic sub-grid modeling Can do much better than deterministic ones *(Buizza et al.*, 1999; *Houtekamer et al.*, 1996)!

Quantitative statements with the help of : $C^{(q)}(\varepsilon_\Lambda^F, \varepsilon_\Lambda^0)$

# Conclusions

- EPS, a radical change in weather forecasting: deterministic framework abandoned for a probabilistic framework!
- However, do our current determinstic models can generate a reliable pdf (Liouville or FP ‘ scenarios ’) ?
- fundamental importance of a wide range of time and space scales, as well as of intensities:
  - no characteristic time of predictability,
  - well defined power-law predictability decay,
  - optimize prediction w.r.t. them ?
- Let ’s talk… and work together !