Background errors in HIRLAM variational data assimilation

Flow-dependent aspects of data assimilation
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Magnus Lindskog[1], Ole Vignes[2], Nils Gustafsson[1], Tomas Landelius[1], Sigurdur Thorsteinsson[3] and other HIRLAM researchers

[1]Swedish Meteorological and Hydrological Institute
[2]met. no
[3]Icelandic Meteorological Office

Structure

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- Background error constraint
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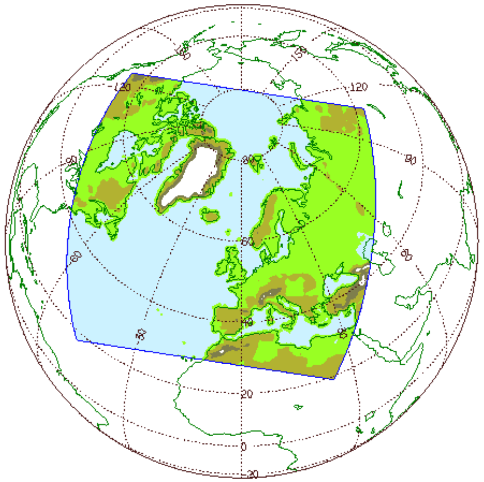
HIRLAM

(HIgh Resolution Limited Area Modeling)



- The Danish Meteorological Institute (DMI)
- The Finnish Meteorological Institute (FMI)
- The Icelandic Meteorological Office (IMO)
- The Irish Meteorological Service (IMS)
- The Royal Netherlands Meteorological Institute (KNMI)
- The Norwegian Meteorological Institute (met.no)
- The National Meteorological Institute of Spain (INM)
- The Swedish Meteorological and Hydrological Institute (SMHI)
- Partly Météo France

Since 2006, close co-operation with ALADIN

- **HIRLAM 3D-Var went operational in 1998**
- **HIRLAM 4D-Var is planed to become operational in 2007**
- **Data assimilation within AROME planed in 2008**

HIRLAM variational data assimilation

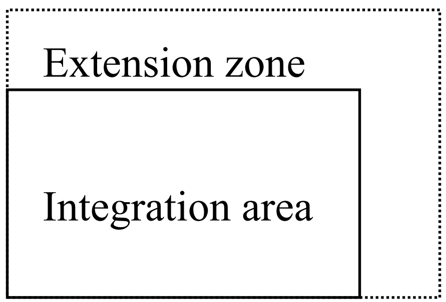
Cost function:

$$\boxed{J = J_b + J_o = \frac{1}{2}\delta x^T B^{-1}\delta x + \frac{1}{2}(Hx^b + H_{tl}\delta x - y)^T R^{-1}(Hx^b + H_{tl}\delta x - y)}$$

$$\delta x = x - x^b = \begin{pmatrix}\delta u & \delta v & \delta T & \delta q & \delta \ln p_s\end{pmatrix}^T$$

$$\chi = U\delta x$$

$$\nabla_\chi J = \frac{1}{2}(\chi)^T(\chi) + \frac{1}{2}(Hx^b + H_{tl}U^{-1}\chi - y)^T R^{-1}(Hx^b + H_{tl}U^{-1}\chi - y)$$



Analytical balance: $\begin{pmatrix}u_{ag} & v_{ag} & T & q & \ln p_s\end{pmatrix}^T$ Statistical balance: $\begin{pmatrix}vor & div^{ub} & T^{ub} & q^{ub} & \ln p_s^{ub}\end{pmatrix}^T$

(J_{LBC} and J_c developed and under evaluation)

Generating background error statistics

The NMC-method

- Error statistics estimated from differences of forecasts valid at the same time (48h-24h or 36h-12h).

Ensemble assimilation

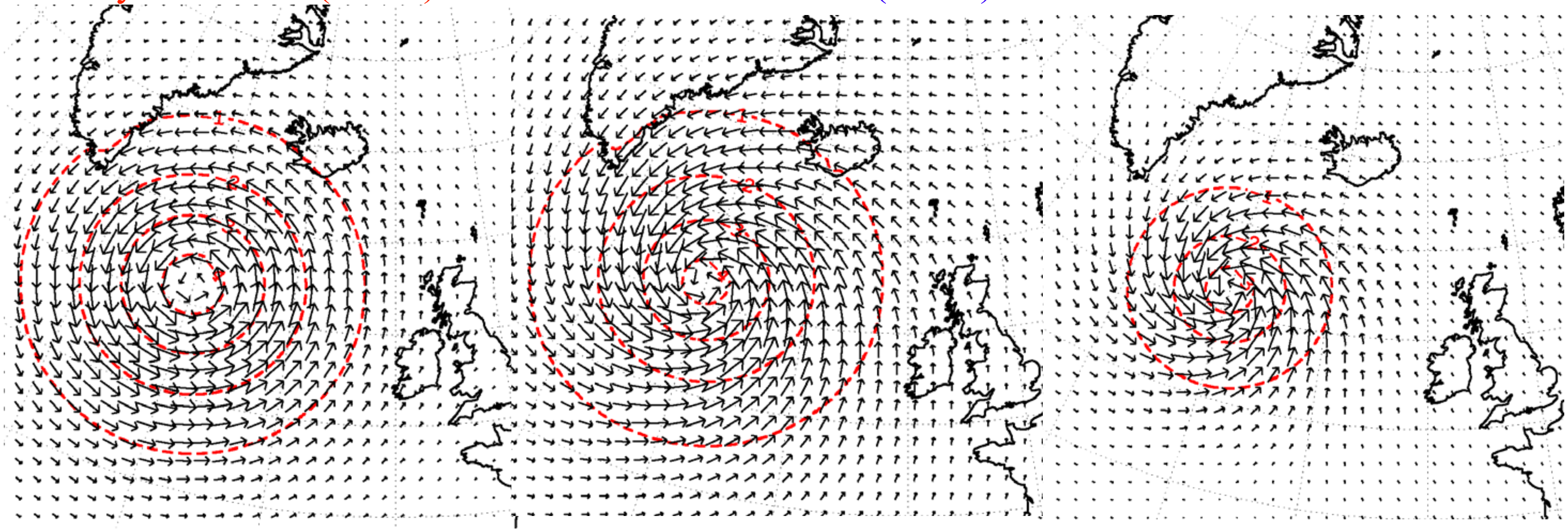
- Error estimated statistics from an ensemble of 6h forecast differences from an ensemble data assimilation experiment with perturbed observations, boundaries, model physics….

Illustration structure functions

Impact of one single surface pressure observation 5 hPa less than the corresponding background equivalent (red: surface pressure, black: winds at lowest mod level)

Horisontal

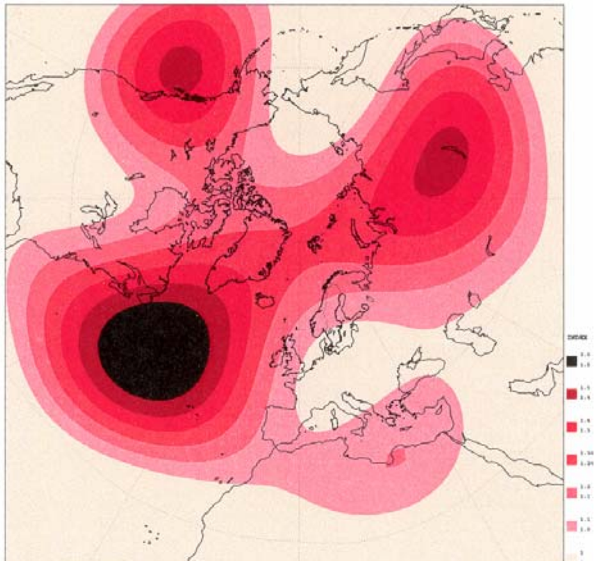
Analytical NMC (48-24) Statistical NMC (36-12) Statistical Ensemble



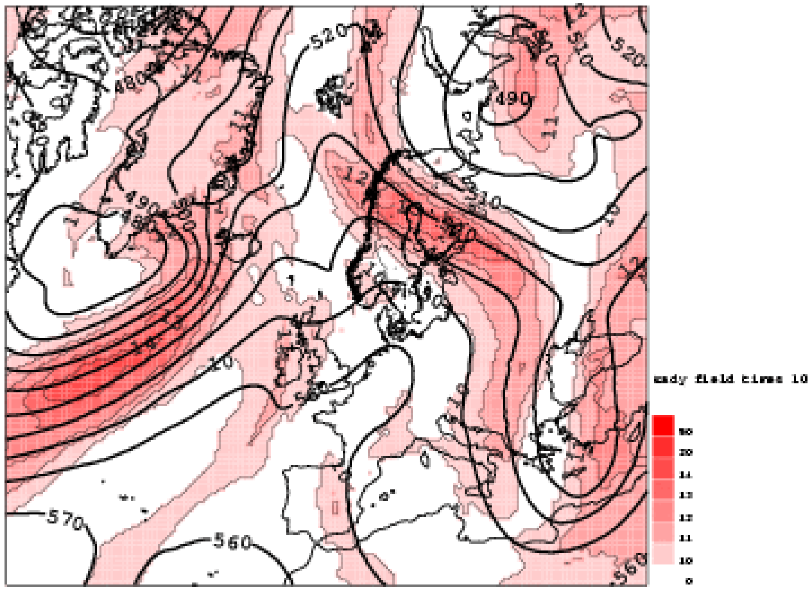
Background error standard deviations

•Background error standard deviations within the assimilation are currently represented by a vertical profile for each type of variable contained within the control vector. Slight seasonal variation.

Horisontal variation



Climatological sigma-b index field (based on observation minus background statistics)



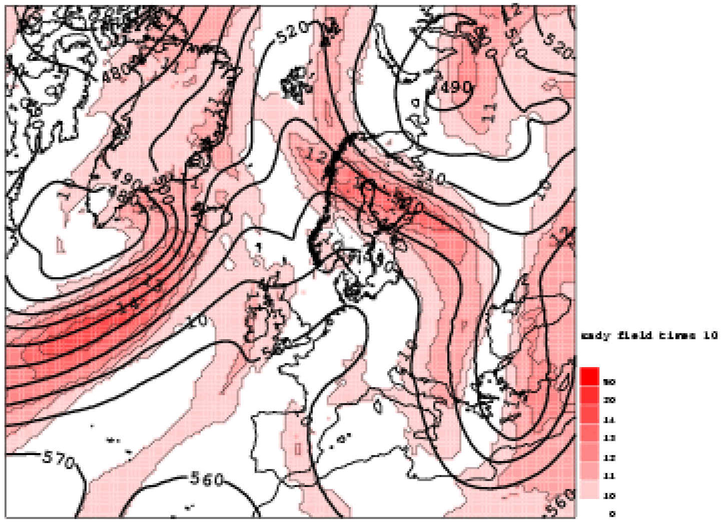
Synoptically varying sigma-b index field (based on Eady index för baroclinic error growth rate).

$$\boxed{\sigma_{BI} = 0.31 f \left|\partial V / \partial z\right| N^{-1}}$$

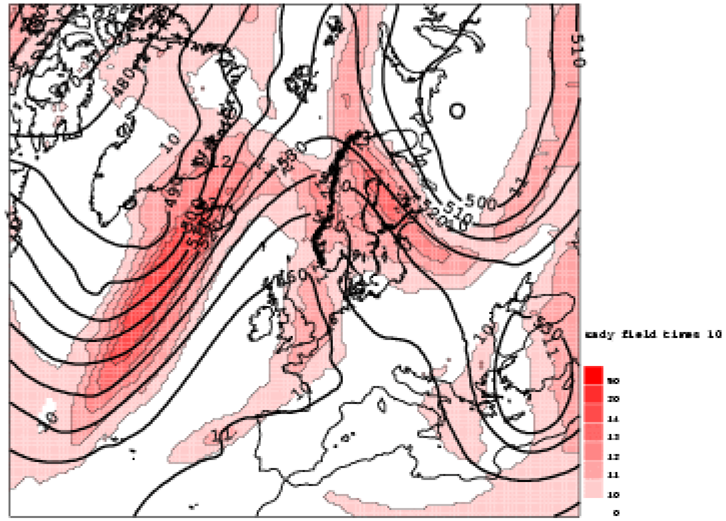
$$(N^2 = g \frac{d \ln \theta_0}{dz})$$

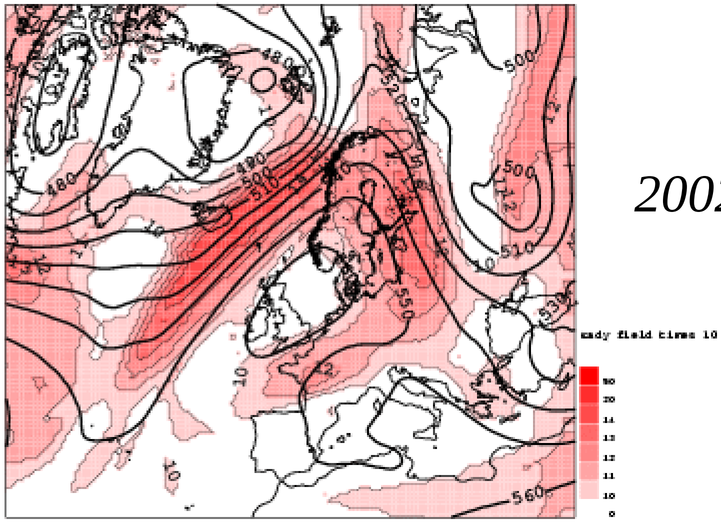
Eady index variation

20020105 18 UTC



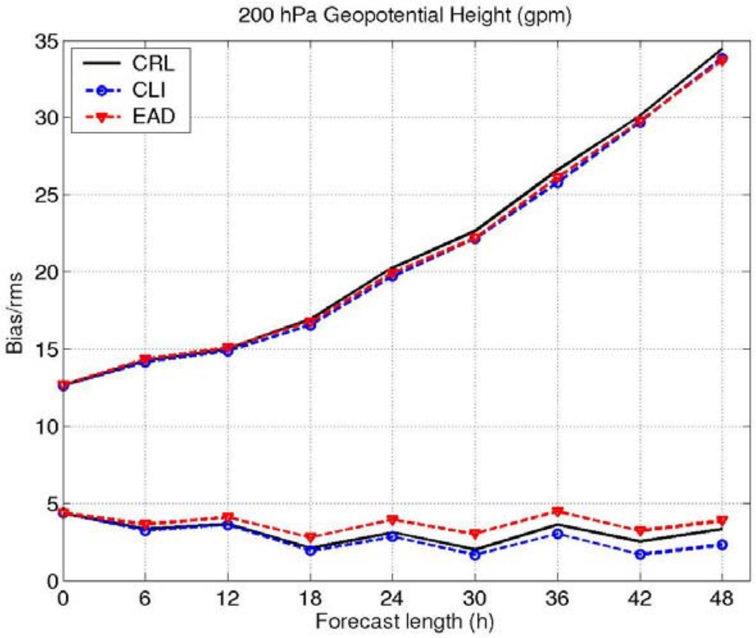
20020106 18 UTC

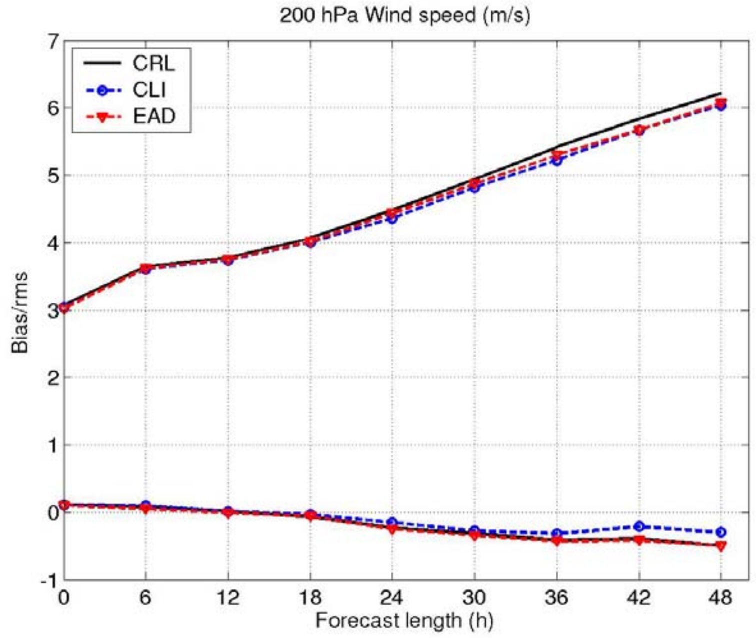


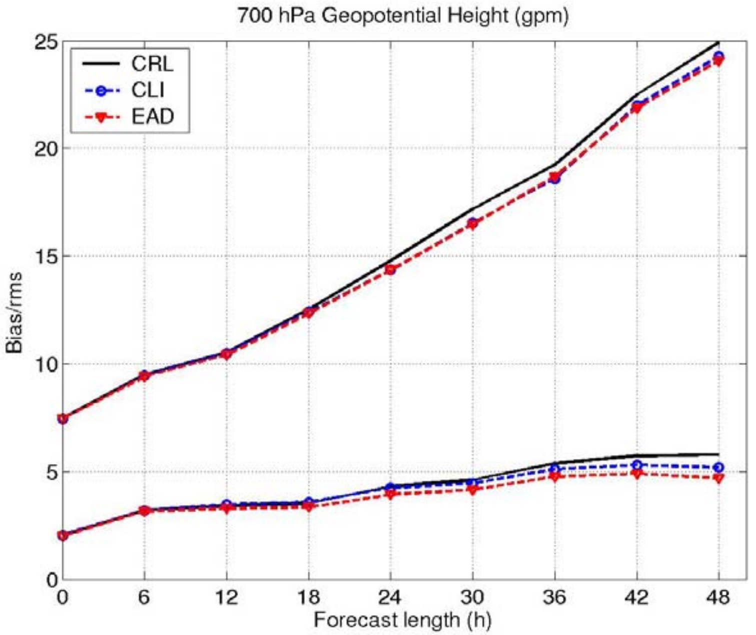


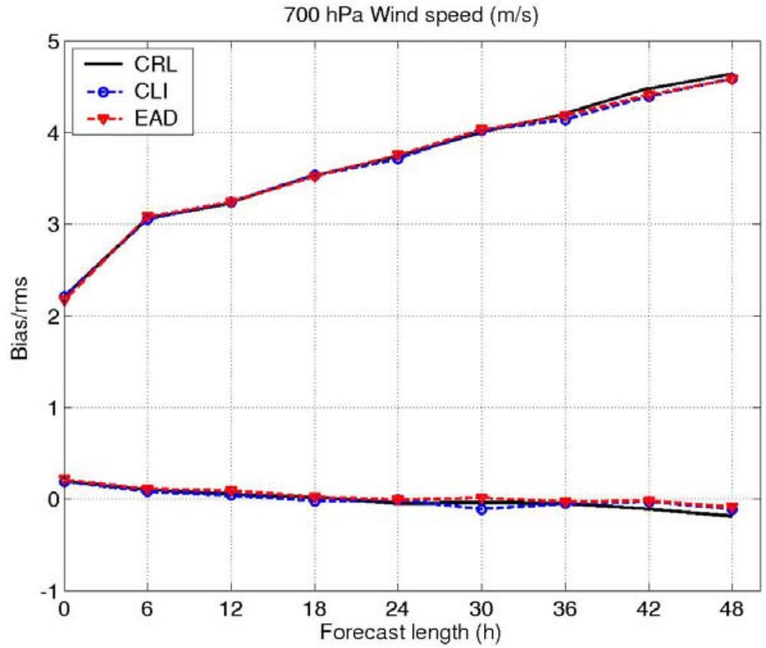
20020107 18 UTC

SCORES from a one month parallel run









Flow dependent background error covariances (ensemble and wavelet representations)

In close collaboration with ALADIN the introduction of a complex wavelet transform (Kingsbury, 2001) is under development to represent the background error covariances.

It is nearly shift invariant and nearly orientation invariant.

Synoptically dependent background errors in the future:

Transform statistics of real time forecast differences calculated from an ensemble of short range forecasts into wavelet space.

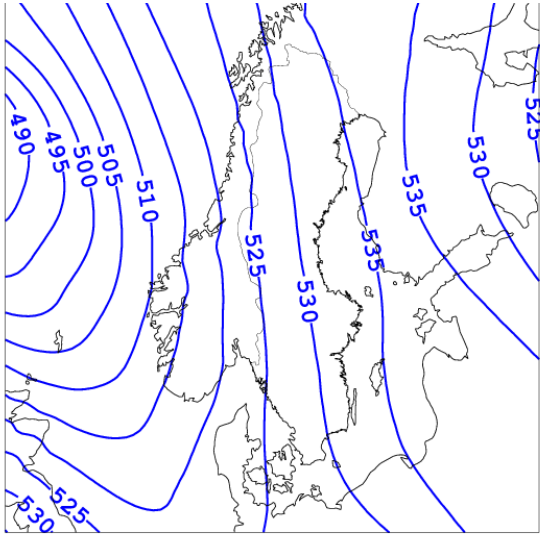
Demonstration:

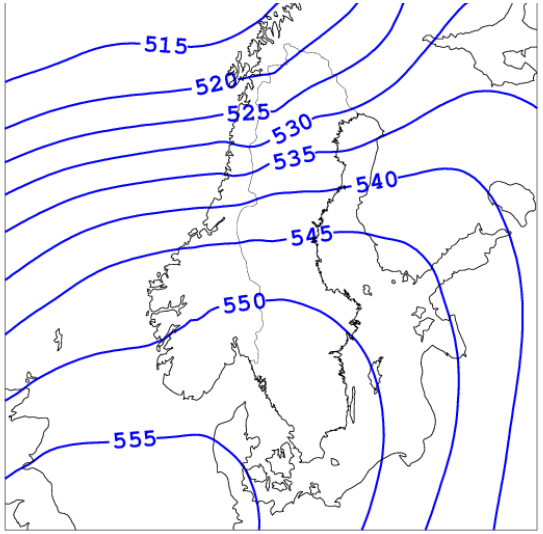
Statistics of 12 h minus 36 h forecasts valid at the same time were calculated for a number of 3 day periods.

Synoptic variation of covariances as calculated from 3 days of 12h minus 36 h forecast differences

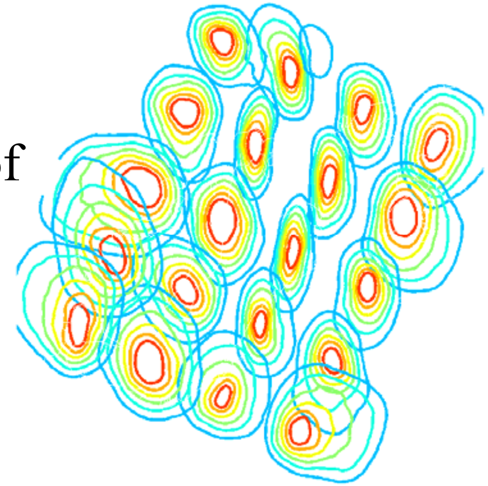
20060116-20060118 **20060116-20060118**

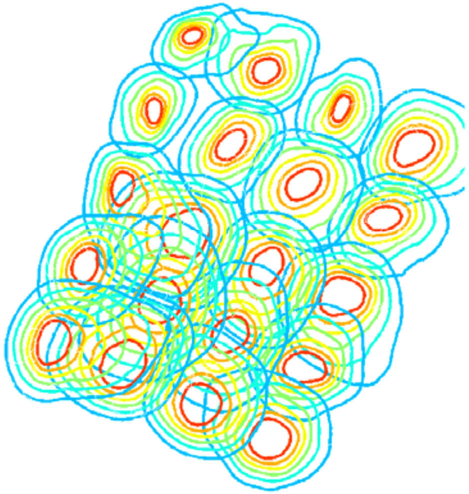
500 hPa geopotential height





Wavelet representation of 500 hPa temperature error covariances





Flow dependent background covariances through non-linear balance and omega equations

Non-linear balance equation on pressure levels:

$$\nabla_p^2 \Phi = \left[f\nabla^2 \psi + + \frac{\partial f}{\partial x}\frac{\partial \psi}{\partial x} + \frac{\partial f}{\partial y}\frac{\partial \psi}{\partial y} + 2\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial y^2} - 2\left(\frac{\partial^2 \psi}{\partial x \partial y^2}\right)^2 \right]_p$$

Tangent-linear version of balance equation on pressure levels:

$$\nabla_p^2 \Phi' = f\left(\frac{\partial v_r'}{\partial x} - \frac{\partial u_r'}{\partial y}\right)_p + \frac{\partial f}{\partial x} v_r' + \frac{\partial f}{\partial y} u_r'$$

$$-2\left(\frac{\partial \bar{v}_r}{\partial x}\frac{\partial u_r'}{\partial y} - \frac{\partial \bar{u}_r}{\partial y}\frac{\partial v_r'}{\partial x}\right)_p + 2\left(\frac{\partial \bar{v}_r}{\partial y}\frac{\partial u_r'}{\partial x} + \frac{\partial \bar{u}_r}{\partial x}\frac{\partial v_r'}{\partial y}\right)_p$$

Tangent-linear version of omega equation on pressure levels:

$$\sigma \nabla_p^2 \omega' + f_0^2 \frac{\partial^2 \omega'}{\partial p^2} = -\nabla_p \cdot Q' + f_0 \frac{\partial f}{\partial x}\frac{\partial u_g'}{\partial p}$$

where

$$Q_x' = -\frac{R_d}{p}\left[\frac{\partial \bar{u}_g}{\partial x}\frac{\partial T'}{\partial x} + \frac{\partial u_g'}{\partial x}\frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{v}_g}{\partial x}\frac{\partial T'}{\partial y} + \frac{\partial v_g'}{\partial x}\frac{\partial \bar{T}}{\partial y}\right]$$

$$Q_y' = -\frac{R_d}{p}\left[\frac{\partial \bar{u}_g}{\partial y}\frac{\partial T'}{\partial x} + \frac{\partial u_g'}{\partial x}\frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{v}_g}{\partial y}\frac{\partial T'}{\partial y} + \frac{\partial v_g'}{\partial x}\frac{\partial \bar{T}}{\partial y}\right]$$

Tangent-linear version of balance and omega equations on HIRLAM model levels

When discretized it reduces to a linear system of equations of the form:

$$Ax = b$$

Introduction in the background error constraint not yet ready.

Weak constraints

Define the residual:

$$r(\delta x) = \begin{bmatrix} r_\Phi \\ r_\omega \end{bmatrix} = \begin{bmatrix} b_\Phi - A_\Phi \delta\Phi \\ b_\omega - A_\omega \delta\omega \end{bmatrix} = L\delta x$$

Add a new penalty term to the cost function:

$$J(\delta x) = J_b + J_o + J_{be},$$

where

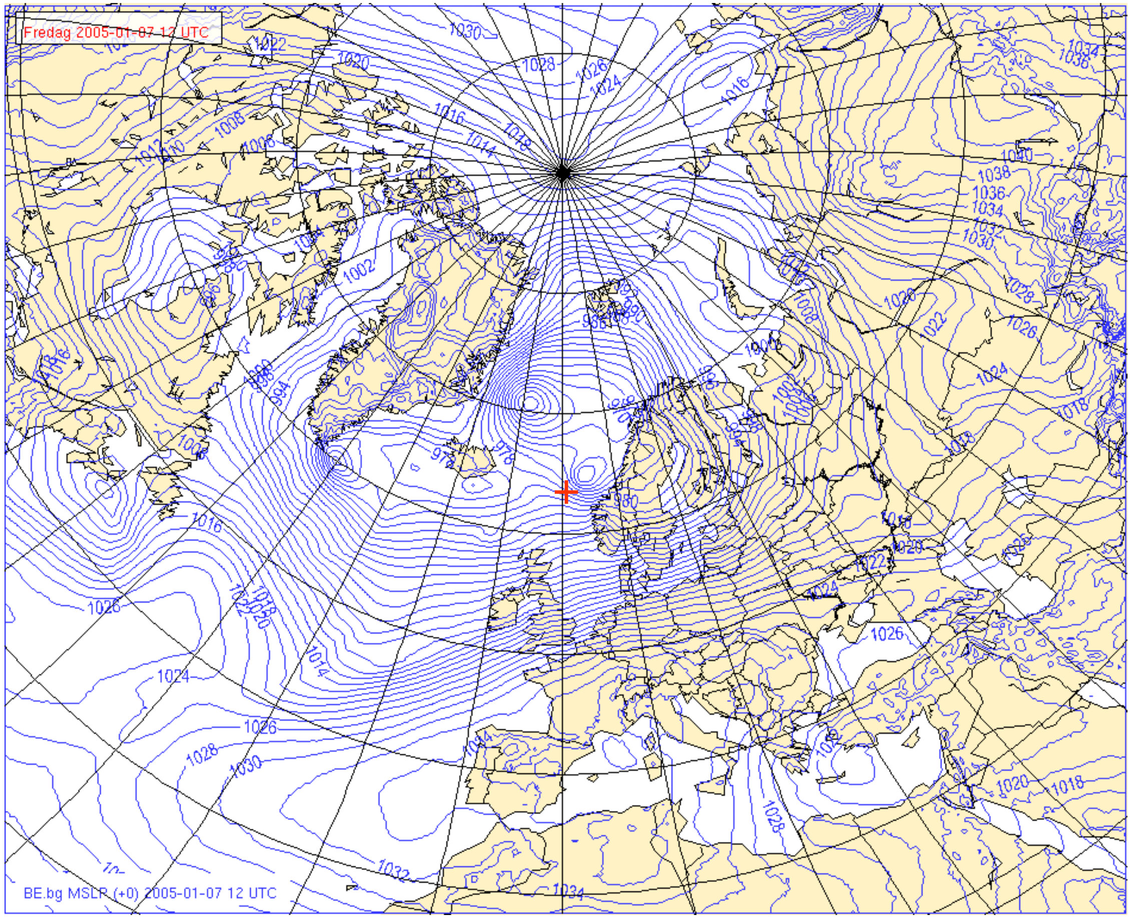
$$J_{be} = \frac{1}{2} r^\top W r = \frac{1}{2} \delta x^\top L^\top W L \, \delta x.$$

So far the weighting matrix W has been diagonal, but the residuals r_Φ and r_ω have been scaled to have similar magnitude.

The quadratic form of the cost function is kept.

Some results

Single obs. experiment, one T increment of -5 °C at 925 hPa, 7/1-2005



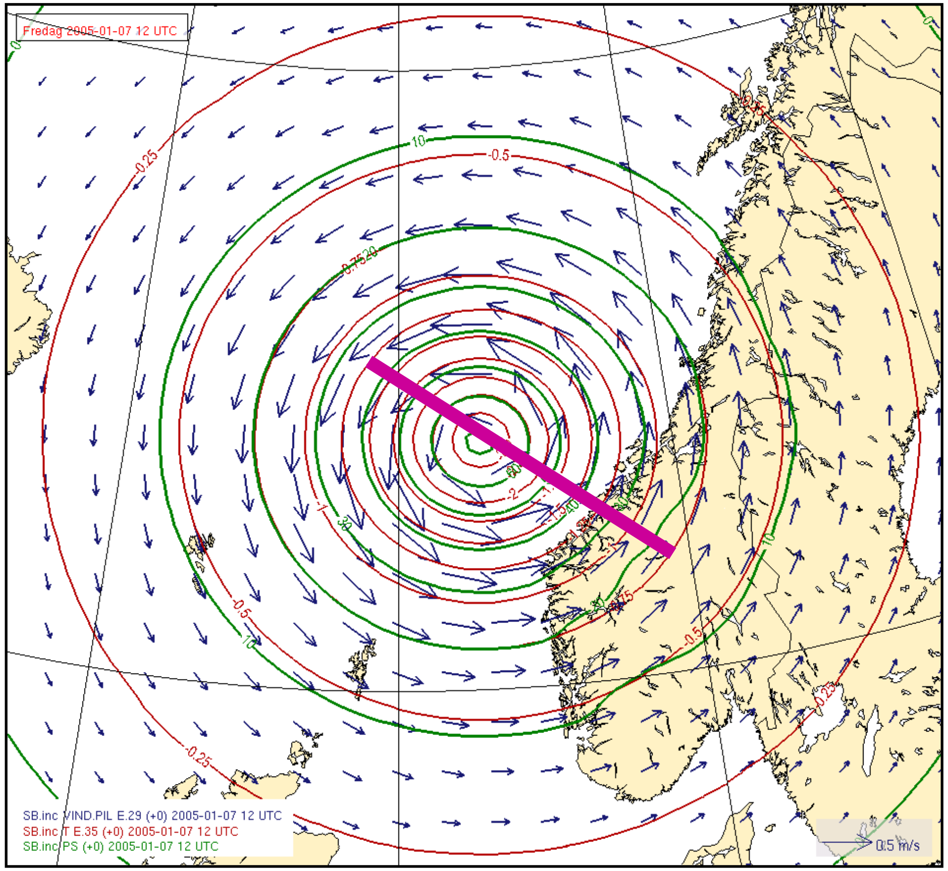
Increments of T, wind, ps

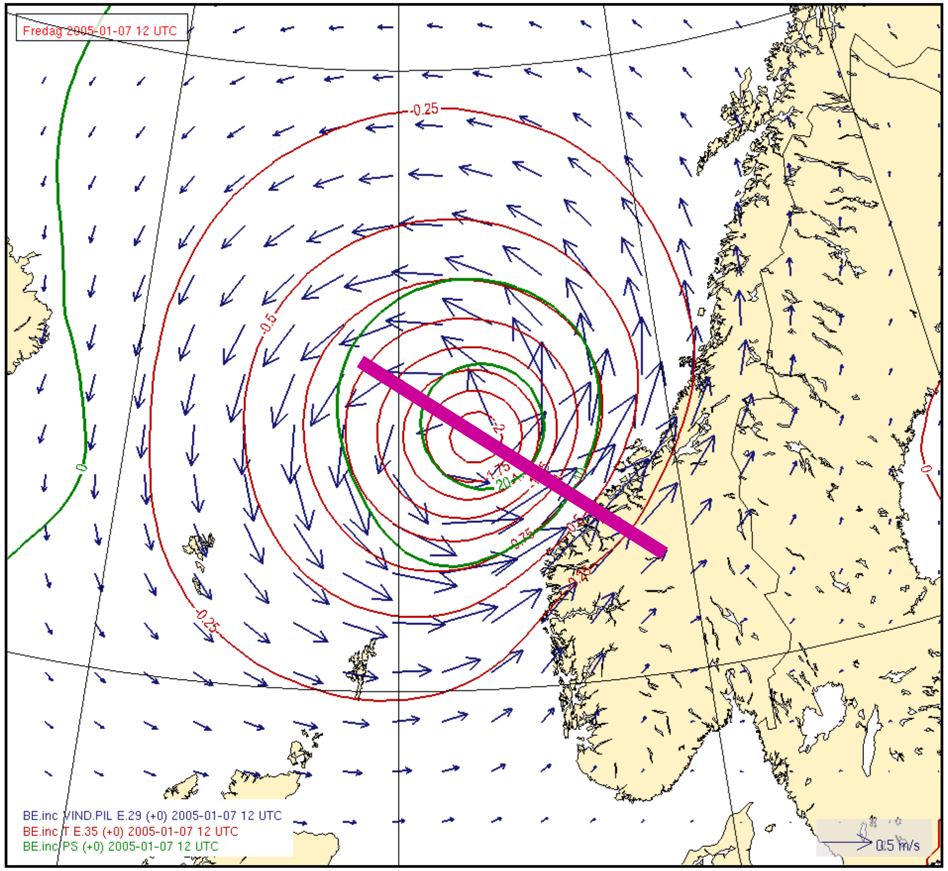
Statistical balance, Jbe = 0

3 iterations

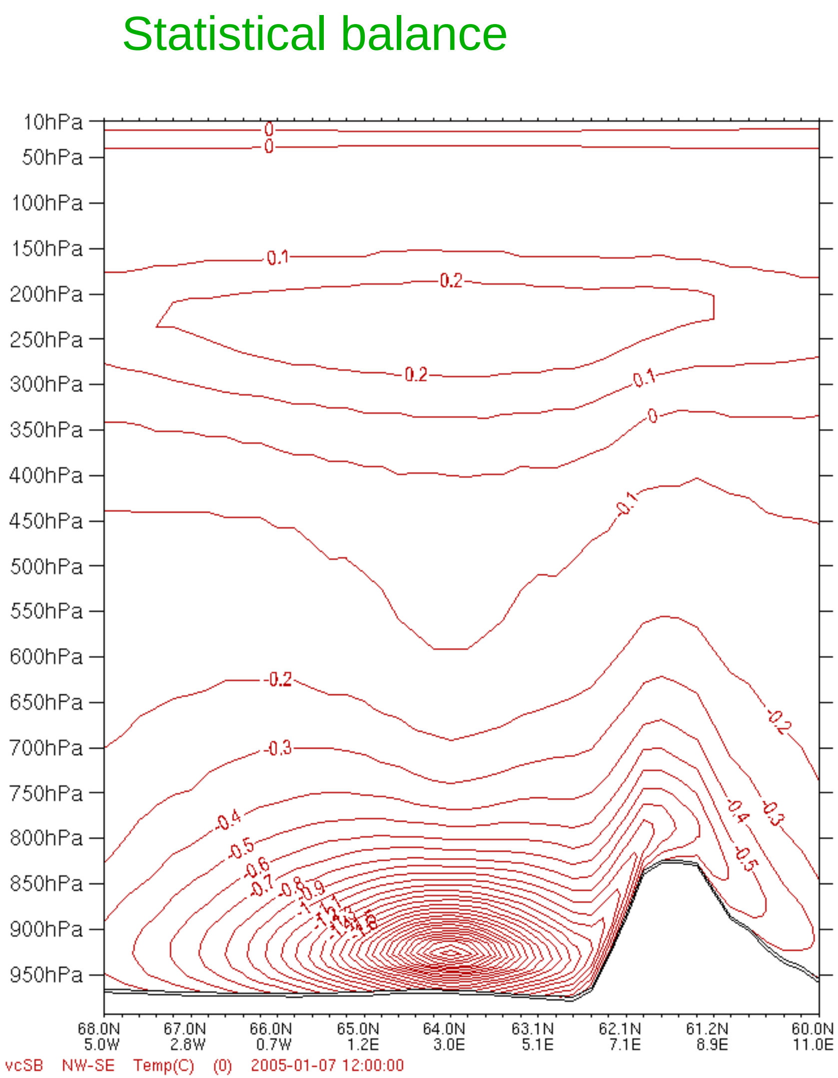
Weak constraints, Jbe weight = 10.

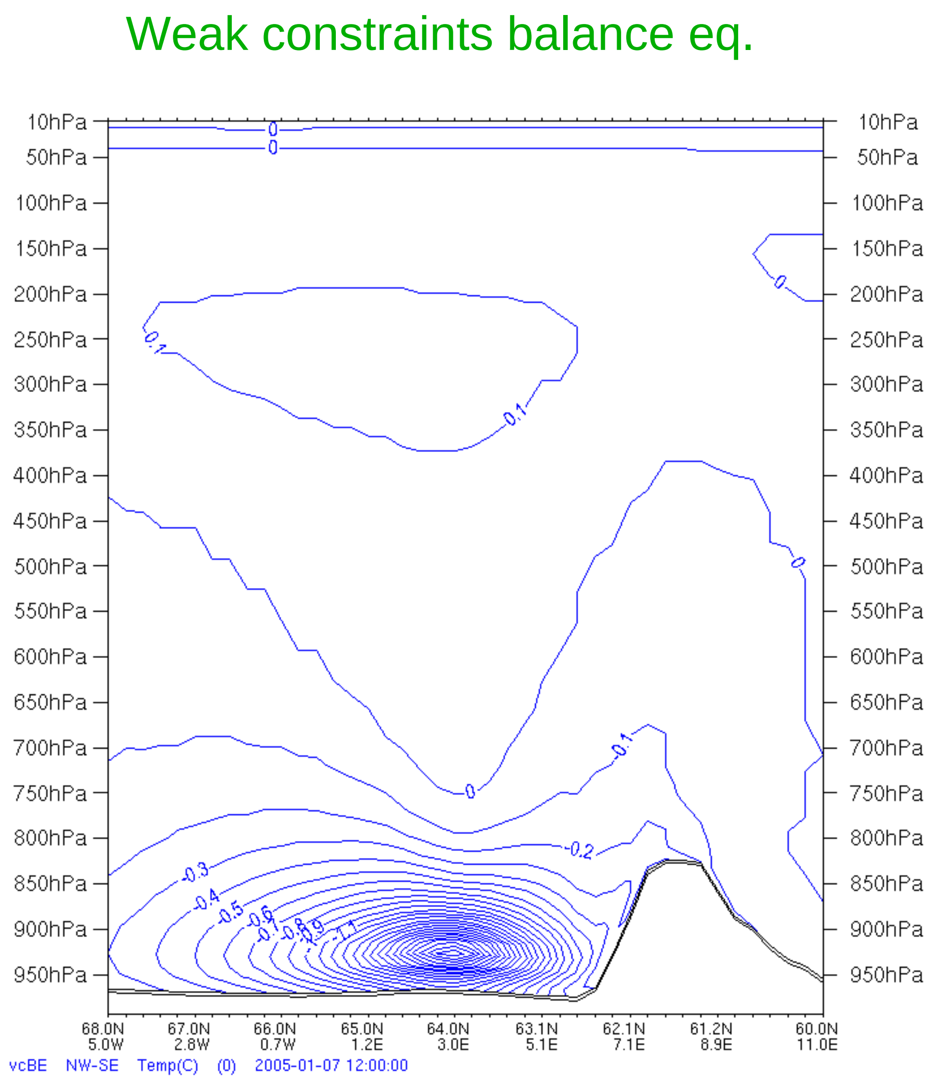
109 iterations





Vertical crossection of T increments

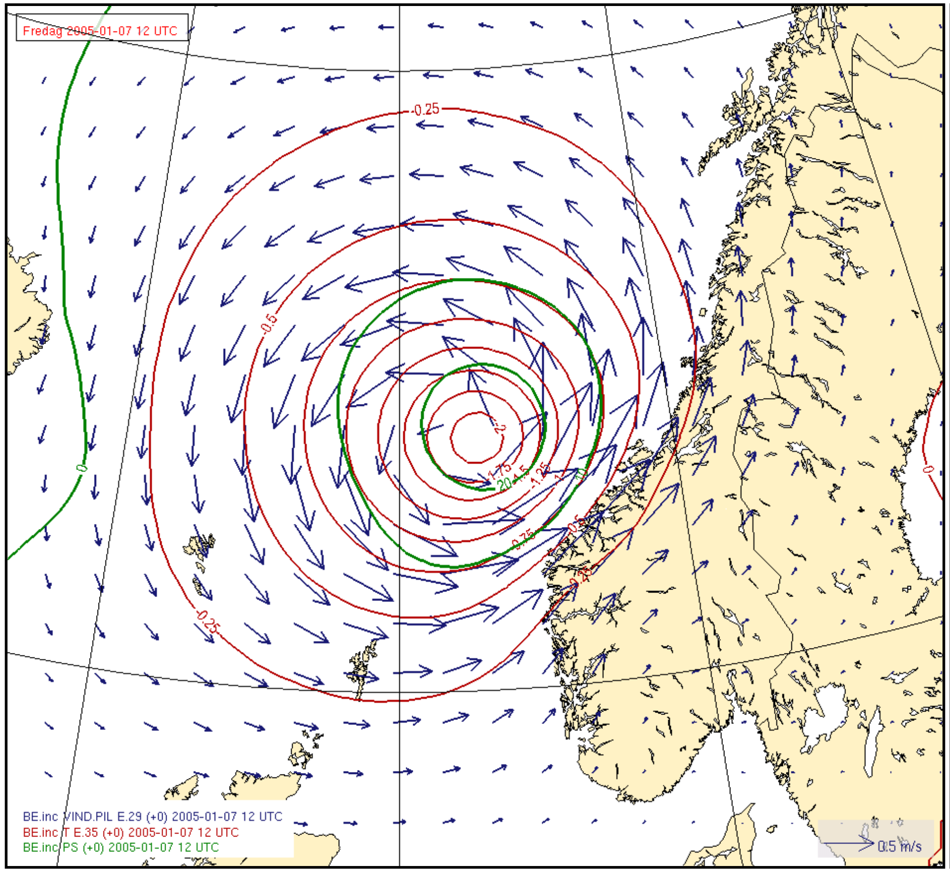




Increments of T, wind, ps

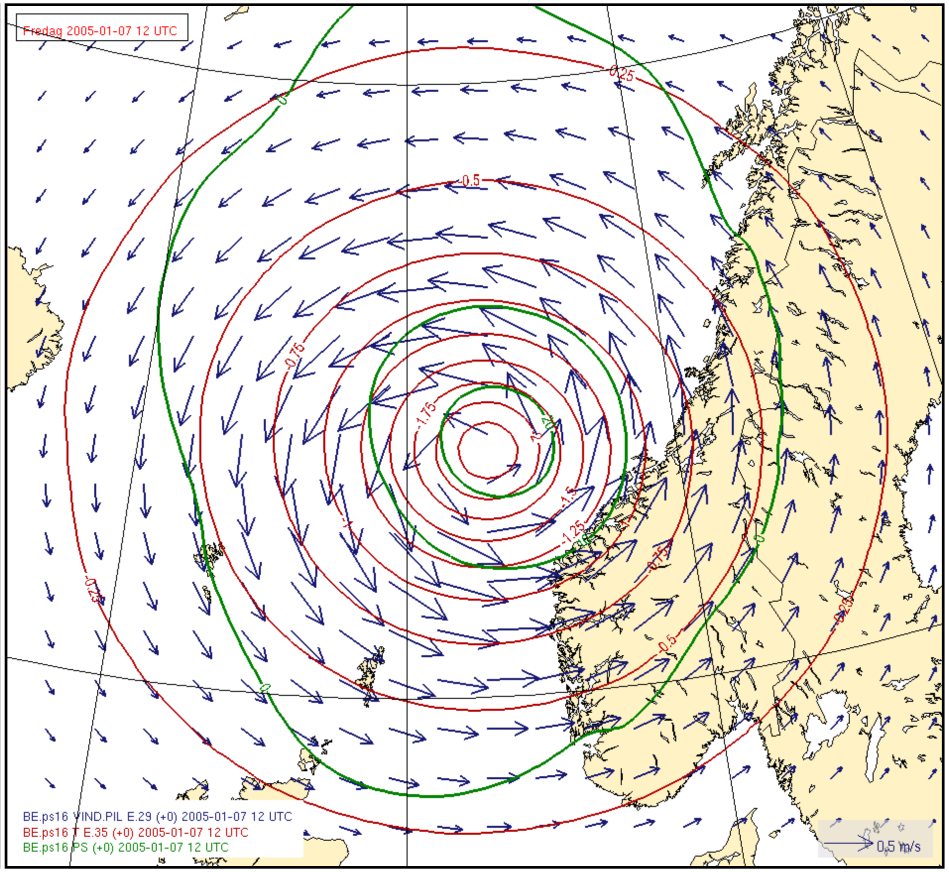
Weak constraints, weight = 10.

109 iterations



Weak constraints, dp/dx = dp/dy = 0.

24 iterations

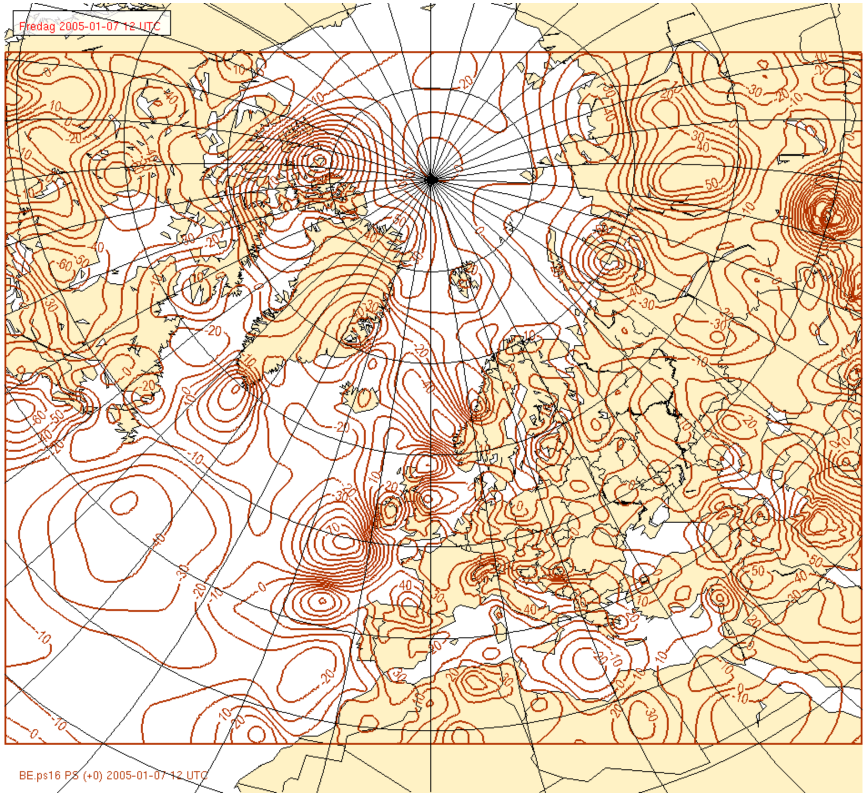


Ps incr. with a full set of observations, 7/1-05 12z

Statistical balance, 22 iterations

Jo = 13226

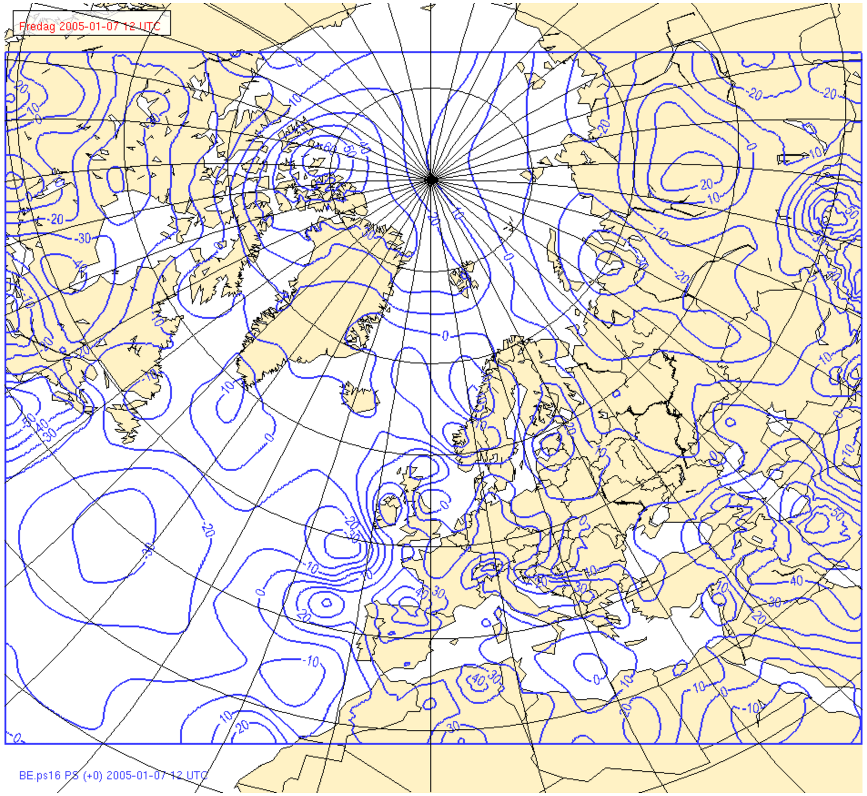
Jb = 1442



Weak constraints, weight 10., 112 iter.

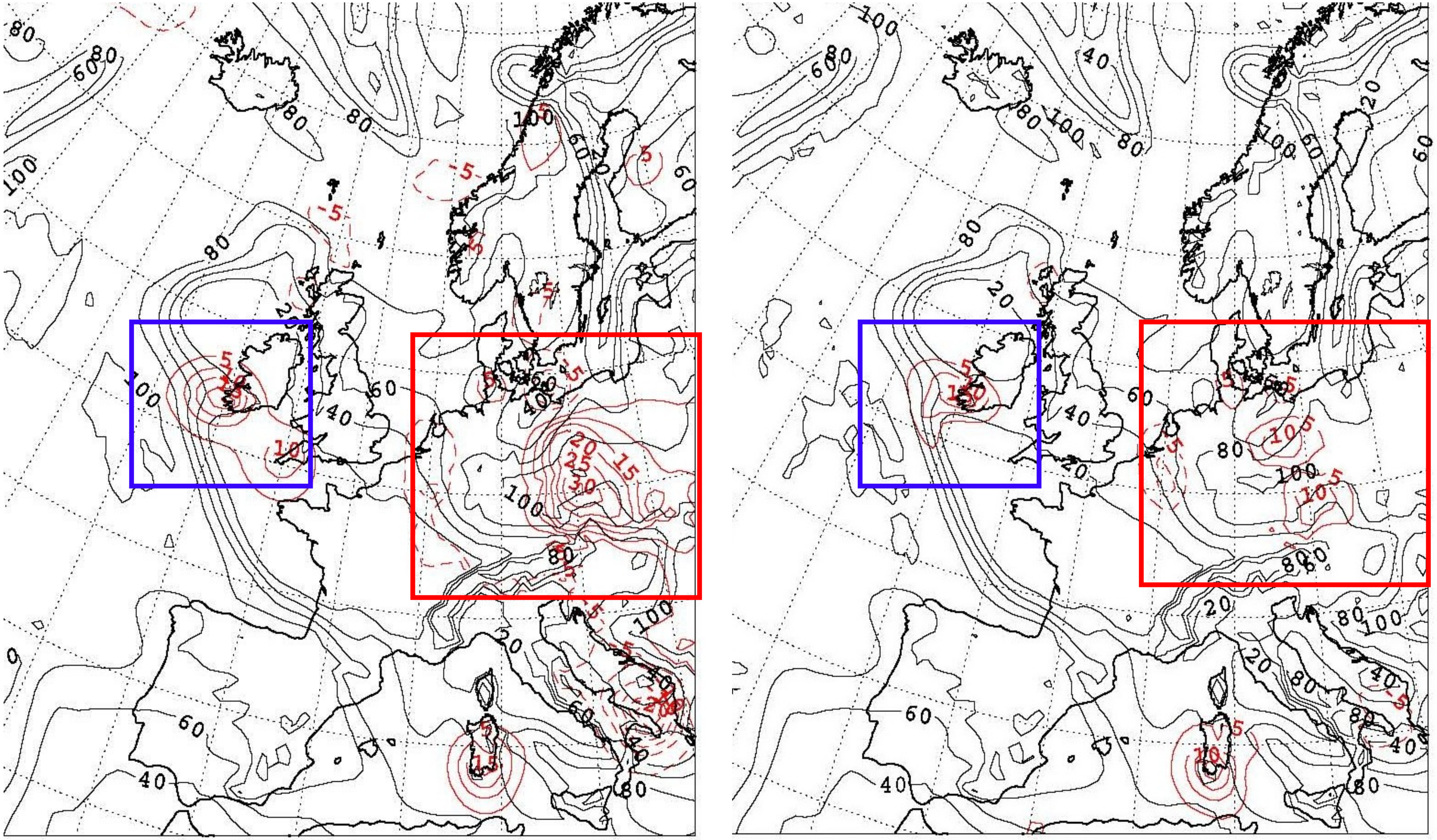
Jo = 13877

Jb = 1146, Jbe = 121



New assimilation control variable for humidity

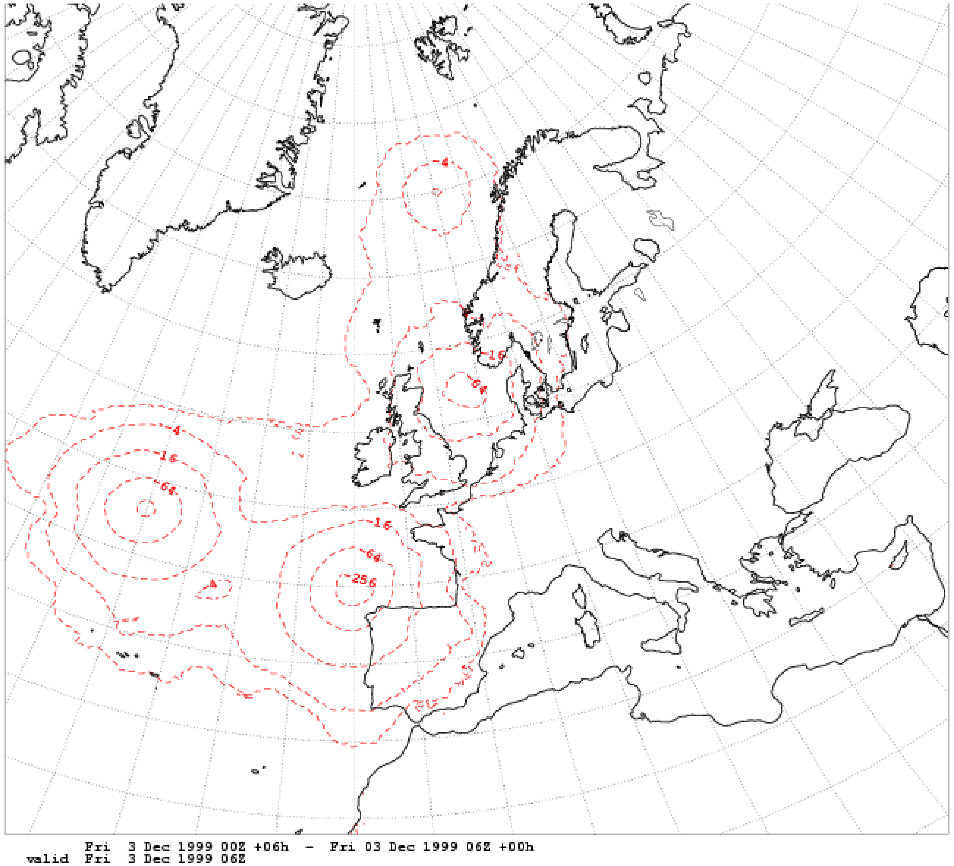
Old formulation δq **New formulation:** $\delta RH^* = \delta RH/\sigma_b(RH_b + 0.5\delta RH)$



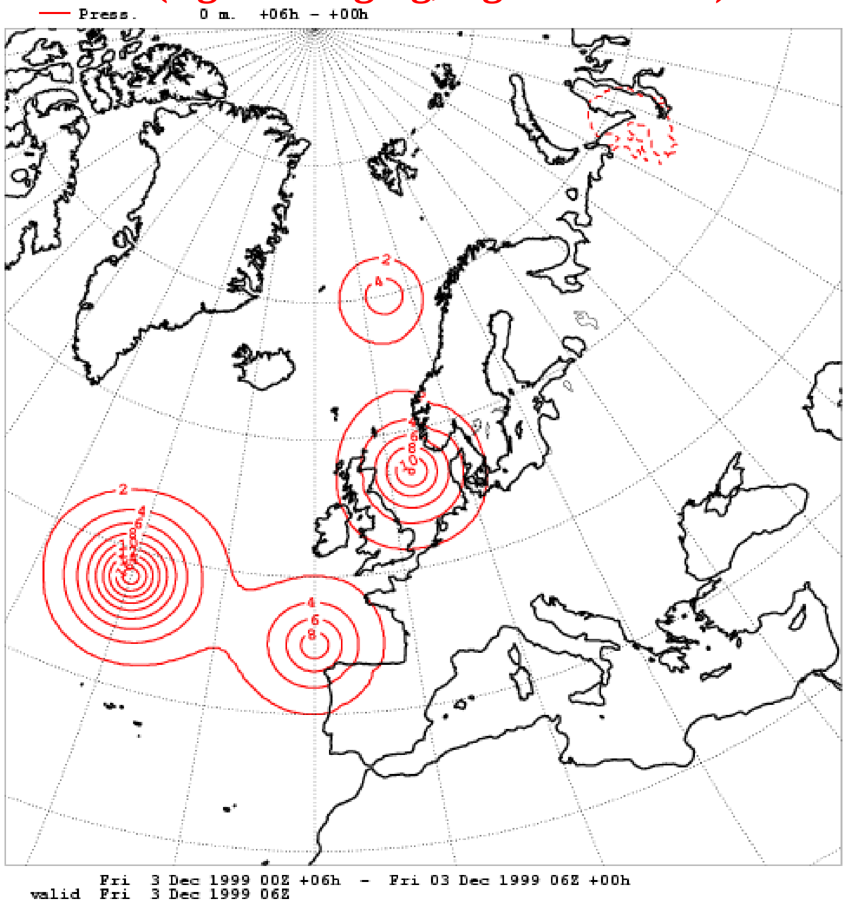
New assimilation control variable for humidity (statistical balance with multivariate humidity)

$\delta RH^* = \delta RH / \sigma_b(RH_b + 0.5\delta RH)$ **Slightly mod. to avoid doublecounting of balances of q with T and p_s**

Assimilation increments due 5 simulated specific humidity observations, 10 g/kg smaller than corresponding background equivalent (sigmao: 1 g/kg, sigmab: const.)



q at 850 hPa (g/kg times 10)



p_s (hPa times 10)

Summary

•Static structure functions have been generated for HIRLAM, using different methods and balances.

•Statistical balance formulation gives best results.

•Eady index based representation of flow dependent background error standard deviations is developed.

•Complex wavelet transform for representation of background error covariances in combination with ensemble assimilation seem promising.

•Interesting results obtained with non-linear balance equation and omega equation on model levels.

•Introduction and further evaluation of new moisture control variable is presently ongoing.