

Humidity-convection feedbacks in a mass flux scheme based on resolved size densities

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1 Introduction

Cumulus cloud populations remain at least partially unresolved in present-day numerical simulations of global weather and climate, and accordingly their impacts on the larger-scale flow have to be represented through parameterization. Various methods have been developed over the years. Perhaps the simplest method is the so-called “bulk approach”, in which only the average properties of a whole ensemble of sub-grid fluctuations are considered. More complex approaches have also been developed, attempting in some way to reconstruct the probability density functions that describe the sub-grid variability by using more degrees of freedom. Examples are statistical schemes and multiple plume approaches.

The ever increasing computational speed and efficiency of supercomputers is driving the application of ever finer discretizations. This creates problems for existing sub-grid schemes in operational circulation models. Ideally, a sub-grid scheme should automatically adapt its impact on the resolved scales to the dimension of the grid-box within which it is supposed to act. It can be argued that this is only possible when i) the scheme is aware of the range of scales of the processes it represents, and ii) it can distinguish between contributions as a function of size. How to conceptually represent this knowledge of scale in existing parameterization schemes remains an open question that is actively researched.

This study reviews a relatively new class of models for sub-grid transport in which ideas from the field of population dynamics are merged with the concept of multi plume modelling. More precisely, a multiple mass flux framework for moist convective transport is formulated in which the ensemble of plumes is created in “size-space”. It is argued that thus resolving the underlying size-densities creates opportunities for introducing scale-awareness and scale-adaptivity in the scheme. In addition, the behavior of a simple implementation of this framework is examined for a standard case of subtropical marine shallow cumulus convection. One of the main questions asked in this study is if a system of multiple independently resolved plumes is able to automatically create the vertical profile of bulk (mass) flux at which the sub-grid scale transport balances the imposed larger-scale forcings in the cloud layer.

2 The framework

At the foundation of the model is the probability density function of an ensemble of cumulus clouds as a function of their size l . The total number of cumulus clouds in the domain number density N can be expressed as an integral over the number density $\mathcal{N}(l)$,

$$N = \int_l \mathcal{N}(l) dl. \quad (1)$$

In principle size l can be defined in more than one way (e.g. [Neggers et al. 2003](#)); what is important is that the resulting density captures a functionality as a function of size in important behavior such as

transport and cloud properties.

Related to (1) is the size density of the associated area fraction a that is occupied by the ensemble cumulus clouds,

$$a = \int_l \mathcal{A}(l) dl = \frac{1}{A} \int_l \mathcal{N}(l) l^2 dl \quad (2)$$

where A is the total area of the modeled domain, which can be a grid-box in a GCM or a simulation domain in an LES.

The final step is to write the turbulent flux $\overline{w'\phi'}$ in terms of size densities, making the mass flux approach and introducing dependence on height z ,

$$a \overline{w'\phi'^a} \approx \int_l \mathcal{A}(l, z) w(l, z) [\phi(l, z) - \bar{\phi}(z)] dl \quad (3)$$

$$= \frac{1}{A} \int_l \mathcal{N}(l, z) l^2 w(l, z) [\phi(l, z) - \bar{\phi}(z)] dl \quad (4)$$

3 Interpretation

The system described above is in essence a so-called ‘‘spectral model’’, as the parameterized quantity (here transport) is formulated as a function of the size of the processes behind it. This in itself is not a novelty; spectral models for cumulus convection have been formulated before (e.g. [Arakawa and Schubert 1974](#)). A key step in the practical application of this class of models is the eventual treatment of the (l, z) fields that appear in (4). Early spectral models still apply some kind of bulk method to parameterize these fields, often for reasons of computational efficiency. This means that assumptions still have to be made on the shape of the underlying distributions; the exact way how this is achieved differs per method.

The novelty of the method described here is that instead of making a bulk assumption the (l, z) fields will be *resolved*, using a rising plume model to independently model multiple parts of the size density. This is illustrated in Fig. 1. The size density is discretized into a histogram, consisting of a limited number of bins. This number should be large enough to resolve the subtle vertical structures in profiles of bulk (mass) flux as seen in LES results, but small enough to still guarantee computational efficiency. The average properties of each bin are estimated using a rising plume model, which is initiated at the surface and is allowed to condensate.

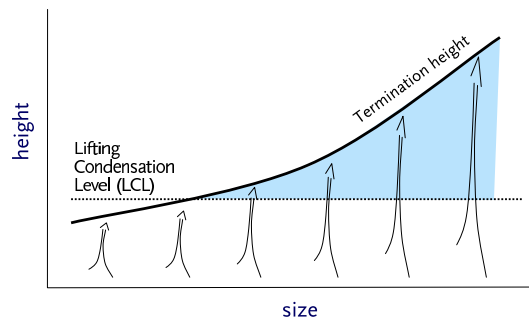


Figure 1: Schematic illustration of an ensemble of plumes each representing a different part of a size density. The Lifting Condensation Level (LCL, dotted line) and termination height (solid line) of the plumes are also shown, for visualization. The area between those lines (shaded blue) represents the height range where the plumes are condensed.

Various versions of such spectrally-resolved convection schemes have recently been proposed (e.g. [Plant and Craig 2008](#); [Wagner and Graf 2010](#)). The aim of this study is to explore the behavior and

inner workings of this class of schemes in more detail, by means of numerical experiments with a single-column model of a subtropical marine shallow cumulus capped boundary layer. However, before performing such experiments, some advantages and disadvantages can already be distinguished from the formulation alone. These will now be shortly reviewed and discussed, as this might aid the interpretation of the numerical results.

The first advantage of a scheme based on resolved size densities is that an assumption on the vertical structure of the bulk (mass) flux, typically made by prescribing a bulk entrainment rate, is no longer required; it is simply obtained by integrating the resolved distribution with size at every height. In operational bulk mass flux models such assumptions on the vertical structure have proven to be a major source of problems, in that i) the vertical structure is typically regime-dependent, ii) the stability of the numerical simulation is often very sensitive to its exact value, and iii) it also significantly affects the climate of the host 3D model in which it is embedded. A potential benefit of a system of multiple independently resolved plumes is that it contains enough freedom to create any vertical profile of bulk (mass) flux; that it will actually find the right one is not trivial.

A second advantage of a scheme based on size densities is that good evidence exists, from both observations and recent LES studies, for the dependence of cumulus cloud properties on their size (e.g. [Dawe and Austin 2012](#); [Boing et al. 2012](#)). In a model framework based on resolved size densities it is relatively straightforward to formulate plume initialization and lateral entrainment as a function of size.

A third advantage is that the independently calculated plumes can interact with each other indirectly through the time-development of the mean thermodynamic state. This potentially introduces population dynamics, in which the behavior of one species (i.e. size) over time can affect the others, and vice versa. Good examples are humidity-convection feedbacks, in which a plume “feels” the humidity transport (or lack of it) by the rest of the ensemble at the previous time-steps.

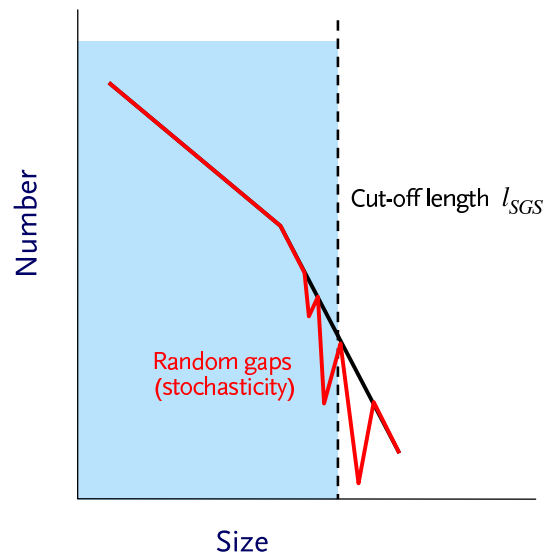


Figure 2: Schematic illustration of the low-pass filtering of a number density.

A fourth advantage is that the system is in principle scale-aware, in that it knows about the distribution of transport over a range of sizes. This implies that in principle scale-adaptivity can be introduced by applying a low-pass filter below a certain cut-off size l_{SGS} that is related to the gridspacing, as schematically illustrated in 2. The work done by all sizes below the cut-off size is then maintained, while the work done by larger sizes is considered to be resolved by the host-model itself. The opportunity also exists to introduce stochastic effects by subsampling the retained PDF at or slightly below the cut-off size.

Many open questions remain. One may ask how many resolved plumes are sufficient to properly represent cumulus transport while still maintaining computational efficiency. One may also wonder how the set of independently resolved plumes will interact and how this will affect model stability; does the system reach a stable and smooth solution for a given set of forcings? A final question concerns the number density, which is a new variable that still requires closure. The remainder of this paper is dedicated to addressing these questions.

4 Experiment setup

4.1 RICO

The convective boundary-layer case simulated in this study was formulated by Working Group I of the Global Energy and Water Cycle Experiment (GEWEX) Cloud System Studies (GCSS, [Browning 1993](#)). The Rain in Cumulus over the Ocean (RICO) case is based on observations made during the field campaign of the same name in 2004, and describes fair-weather Caribbean shallow cumulus. The so-called “composite case” is described in great detail by [van Zanten and Co-Authors \(2011\)](#). The simulation last 72 hours, during which the cumulus-topped boundary layer gradually deepens from 1 km to about 2.5 km, while cloud base stays more or less constant at about 600 m. Prescribed advective forcings are used that are constant with time.

4.2 EDMF

To test the model based on resolved size densities as described in Section 2 it is embedded in the Eddy Diffusivity Mass Flux framework (EDMF, see (e.g. [Siebesma et al. 2007](#); [Negggers et al. 2009](#))) as implemented in the single-column model of the Regional Atmospheric Climate Model (RACMO). In the EDMF approach the turbulent/convective vertical flux is partitioned into a diffusive part and an advective part,

$$\overline{w'\phi'} = -K \frac{\partial \overline{\phi}}{\partial z} + \sum_i M_i [\phi_i - \overline{\phi}] \quad (5)$$

where K is the eddy diffusivity coefficient and M is the volumetric mass flux. Subscript i indicates the properties of the i -th bin in the discretized size density, and $I = 10$ is the total number of bins.

The rising plume model used to resolve the (l, z) fields has the standard form as proposed by [Siebesma et al. \(2007\)](#). Some assumptions need to be made on plume initialization and lateral mixing. Similar to [Plant and Craig \(2008\)](#) the lateral entrainment rate ε_i is assumed to be a function of the size of the plume,

$$\varepsilon_i = \frac{1}{l_i} \quad (6)$$

As described by [Negggers et al. \(2009\)](#) the initial properties of the plumes are assumed to scale with the width of the joint-PDF in ϕ and w , as derived from surface similarity theory. Plumes are allowed to generate condensate when their total specific humidity exceeds the saturation specific humidity. Finally, for simplicity, the plumes are assumed not to generate precipitation.

The strong assumption that plumes of a different size have a different thermodynamic and kinematic state from the start is made here purely out of convenience, as this is an existing feature in our implementation of the EDMF approach. However, note that nothing prevents the use of a different, more sophisticated plume initialization scheme in the model framework described in this study. More research is needed to this purpose, especially in the dry sub-cloud layer where the nature of the turbulence is different (i.e.

more isotropic) compared to the cloud layer. Recently published research seem to support the dependence of cumulus cloud properties on size; perhaps not on sub-cloud parcel-scale (Romps and Kuang 2010) but at least on the cloud-scale (Dawe and Austin 2012; Boing et al. 2012).

4.3 Number density

Closure for number density still needs to be defined. Two ways will be discussed and illustrated with numerical experiments in the next section.

5 Results

5.1 A prescribed number density

In this section results are presented of experiments with a simplified setup, with the sole purpose of obtaining insight into the basic behavior of the resolved ensemble of plumes. To this purpose the number density is prescribed, using a power-law functionality (Cahalan and Joseph 1989),

$$\mathcal{N}(l) = al^b. \quad (7)$$

where a and $b = -1.9$ are constants of proportionality based on values observed in nature (e.g. Benner and Curry 1998) and diagnosed in LES studies (e.g. Neggers et al. 2003). Constant a is normalized in such a way that the total area covered by the ensemble covers 10 % of the domain, an assumption that is part of the EDMF approach. The size density is bounded at the upper end by an imposed maximum of 1 km, which is loosely based on the cloud properties as seen in LES simulations of RICO. With $I = 10$ this means that the scheme works with a histogram with a fixed bin-width of 100 m.

Figure 5 shows the results for the 72-hour RICO simulation. The time-height contour plot shows that the model reproduces the gradually deepening cloud layer as was diagnosed in LES (not shown here). An attractive aspect is that the solution behaves quite smoothly in time. The next panels evaluate the SCM result against LES for three cloud-related variables. Satisfactory performance is reported on the amplitude and vertical structure of mass flux, cloud fraction and cloud condensate. For cloud fraction and mass flux the maximum at cloud base and the “s-shaped” vertical structure above that are typical of shallow cumulus are reproduced. The representation of cloud condensate in particular has much improved compared to the setup of EDMF using a single moist plume (not shown). A minor negative point is that the cloud top height seems to be somewhat overestimated; this might simply be due to the absence of precipitation in the model.

These encouraging results suggests that an ensemble consisting of 10 independently calculated plumes is already sufficient to resolve the delicate vertical structures of cloud and transport related variables typical of the shallow cumulus cloud layer. This result justifies further investigation of the behavior of the population of plumes. The last panel of Fig. 5 shows a breakdown of the total profile of total mass flux into contributions by separate sizes. Small sizes contribute significantly just above cloud base, and are responsible for creating the local maximum in mass flux.

Perhaps the most surprising result from an engineering point of view is that the significantly larger number of free model variables (compared to a single plume setup) has not lead to unstable behavior, but instead is able to reach a stable solution that smoothly varies in time. This implies that some negative feedback mechanism is active that keeps the system under control, and prevents it from collapsing into chaotic behavior. As we use an ensemble of plumes, this means that somehow the independently calculated plumes “feel” the impact of the work done by the other plumes.

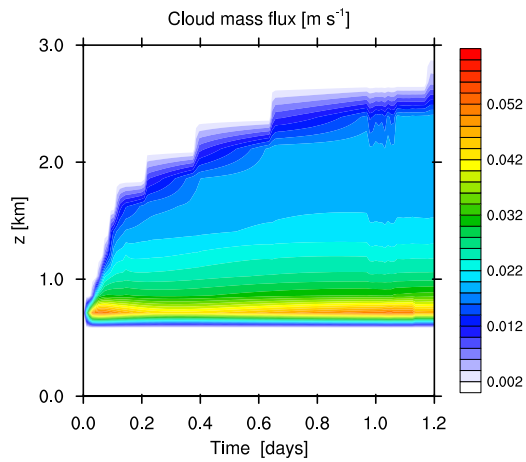


Figure 3: Time-height contour plot of the total volumetric mass flux by those plumes of the ensemble that carry condensed water.

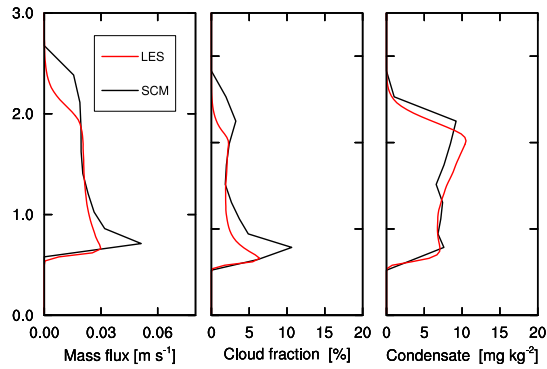


Figure 4: Vertical profiles of a) cloudy mass flux, b) cloud fraction and c) total condensate at $t=24\text{hr}$ after initialization. LES results (red) are included for reference.

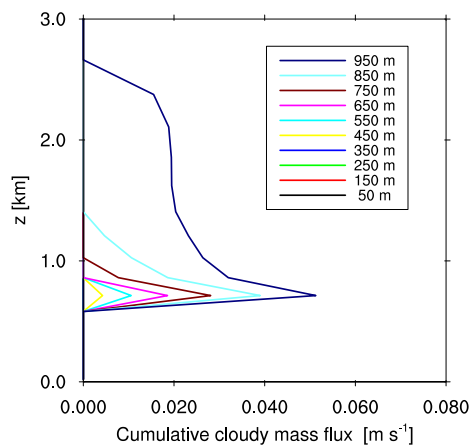


Figure 5: A breakdown of the cloudy mass flux transport as a function of size. Each line represents a cumulative, or an integral of the mass flux density with size, with each color indicating a different maximum.

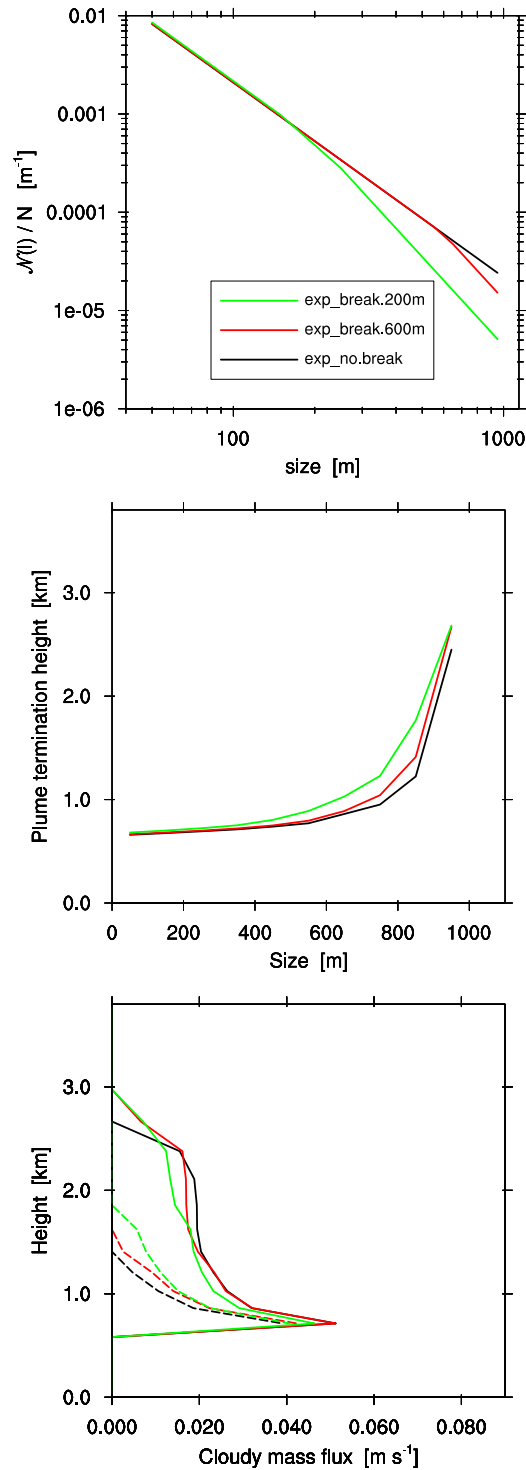


Figure 6: Sensitivity test on the role of the number density $\mathcal{N}(l)$. Shown are a) the number density, b) the plume termination height as a function of size, and c) the cloudy mass flux profile integrated over the whole ensemble. Each color refers to an experiment with a different number density. In the last panel the contribution by all sizes smaller than the top size is indicated by the dashed line.

To get insight into this behavior two additional experiments are now performed in which a scale-break is imposed at $l = 600$ m and $l = 200$ m. This in effect imposes a smaller number of larger sized plumes relative to the smaller ones, as shown in Fig. 6 a. The impact on the behavior of the simulation is shown in Fig. 6 b and c. Although the shape of the total mass flux profile does not change much, the contribution by the smaller sizes increases when larger sizes are suppressed. This is most noticeable in the middle of the cloud layer. The smaller sizes are also able to rise to greater heights.

This behavior can be explained as follows. Prescribing a reduced number of larger plumes lowers their relative contribution to transport. As a result, the instability that was previously overturned by the larger sizes remains, as expressed by a different thermodynamic state. The smaller plumes feel this remaining instability, and will rise further as a result; this is testified by their slightly raised termination height. They thus automatically adapt, and by increasing their contribution to transport help to eventually remove the remaining instability. In effect this mechanism thus represents a negative feedback mechanism between the thermodynamic state (of which humidity is a part) and convection, that helps to make the system quickly converge to a stable solution. This can also be phrased as follows; the smaller sizes “fill the gap” left by the larger sizes and help to recreate the unique vertical profile of bulk transport that is required to counter the prescribed larger-scale forcings.

5.2 Interactive number density

The results presented so far were obtained using a prescribed number density. However, ideally the model should be able to generate its own number density. One way of achieving this is explored here. Suppose $E(l)$ is the vertically integrated kinetic energy of a plume of size l . Then the number of clouds of size l can be expressed as the ratio of the energy of all plumes at that size $E_{tot}(l)$ to that of a single plume,

$$\mathcal{N}(l) = \frac{E_{tot}(l)}{E(l)} \quad (8)$$

The change of total energy $E_{tot}(l)$ with time could be estimated from a budget equation,

$$\begin{aligned} \frac{\partial E_{tot}(l)}{\partial t} &= P(l) - D(l) \\ &\quad - \int_m T(l,m) dm \\ &\quad + \int_m T(m,l) dm, \end{aligned} \quad (9)$$

where $P(l)$ is the production at size l (the “work function”), $D(l)$ is the viscous dissipation at size l , and $T(l,m)$ is the energy transfer from size l to size m .

The terms on the right hand side containing T enable direct interactions between different sizes in the ensemble, and so introduce population dynamics. While the third term is a sink term, representing all energy that plumes of size l lose to the rest of the size-spectrum, the fourth term is a source term, representing the energy that size l receives from the rest of the spectrum. Note that this system in principle allows interactions between all sizes, from local to broad-band, from up-scale to down-scale. However, perhaps the simplest possible transfer model is the local down-scale energy cascade, which could be formulated as follows,

$$T(l, l-dl) dl = \frac{E(l)}{\tau}, \quad (10)$$

where τ is a relaxation time-scale typical of this process. Although being perhaps more applicable to dry mixed-layer turbulence, it is still instructive to explore the model behavior in this setup.

To examine the behavior of the interactive number density model it is implemented into the scheme as tested in the previous Section. The rising plume model is used to estimate i) the energy of a single plume

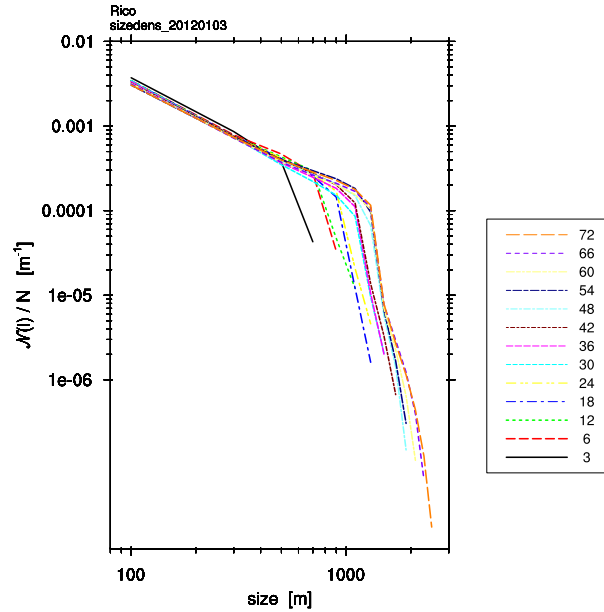


Figure 7: Log-log plot of the number density at various moments during a 72 hr simulation of the RICO case with the interactive number density setup.

$E(l)$, by vertically integrating the vertical velocity profile, and ii) the work function $P(l)$, by vertically integrating the buoyancy flux. The dissipation term $D(l)$ can be modeled as an inverse function of size l . Finally, the maximum size of the modeled size density is assumed proportional to the total boundary-layer depth.

Preliminary results for the RICO case are shown in Fig. 7. The scheme produces number densities that have a power-law shape and include a scale-break, the size of which grows with time. Such distributions have also been observed in nature and in LES (e.g. Benner and Curry 1998; Neggers et al. 2003). The emergence of these features in this simulation can be understood by interpreting the model characteristics. The power-law behavior is explained by the fact that energy transferred down-scale is there distributed over plumes that contain less kinetic energy individually, resulting in a higher number. The scale break size coincides with the size above which condensed plumes start to become positively buoyant in the cloud layer due to latent heating (not shown). This phase change boosts their individual kinetic energy, so that less plumes are required to make up a given total energy E_{tot} .

6 Discussion and conclusions

First steps are made in making an existing mass flux scheme scale-aware and scale-adaptive. To this purpose an existing bulk plume model is equipped with knowledge of the range of scales of the processes that it represents. This is achieved by explicitly modelling individual segments of the size density by means of a rising plume model. In this model the lateral entrainment is a function of the size of the plume. Numerical simulations of Tradewind cumulus with a single-column model illustrate the benefits of this approach;

1. A limited number of plumes is already sufficient to obtain a numerically stable solution while still being computationally efficient;
2. The scheme reproduces the delicate vertical structures of convective mass flux transport, clouds

and condensate within the cumulus layer;

3. This is achieved by means of feedbacks between convective plumes of different sizes through the environmental thermodynamic state;
4. An experimental model for the number density is capable of generating power-law distributions as observed in nature.

The main purpose of this study is to illustrate the above points, which justifies the use of a simplified model setup. However, many assumptions, especially concerning the plume initialization and budget, still lack thorough support by observations or LES results, motivating further research. In addition, the experimental down-scale cascade model of the interactive size density as tested in Section 55.2 is only a first step; the framework is in principle flexible and generally applicable enough to accommodate other modes of energy transport between sizes, such as up-scale energy transport as perhaps more applicable in deeper cumulus convection. Finally, an interesting next step forward would be to learn about the behavior of this scheme when embedded in a 3D model, including a scale-adaptive low-pass filtering of the parameterized size density.

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