Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Representation of model error using stochastic equation with flow-dependent spatial and temporal correlations and noise amplitude

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Motivation

- In Kalman Filter based DA, the weights for the interpolation between the observations and the first guess are inversely proportional to the corresponding uncertainties (errors). The uncertainty of the first guess combines the propagated error of the analysis and the model error. An estimate of the model error is needed in order to give an appropriate weight to the first guess.
- The end-users should be provided with the information how reliable/uncertain the forecast is.
- If the perturbations are chosen correctly, the ensemble mean can (should) be better then the deterministic forecast.

A model for the model error

$$\frac{\partial \phi}{\partial t} = \left[\frac{\partial \phi}{\partial t}\right]_{det} + \eta_\phi(t)$$

ϕ – prognostic variables (T,q,U,V)

$\eta_\phi(t)$ – corresponding model error field

$$\frac{\partial \eta_\phi(x,t)}{\partial t} = -\gamma_\phi \eta + \gamma_\phi {\lambda_\phi}^2 \nabla^2 \eta + \sigma_\phi \xi(x,t)$$

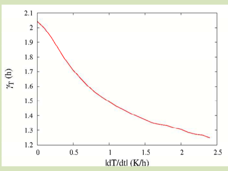
persistence in time, $^1/_{\gamma_\phi}$ – correlation time scale	diffusion establishes spatial correlations (governed by λ_ϕ)	random component, $\xi(x,t)$ – Gaussian white noise, σ_ϕ – standard deviation

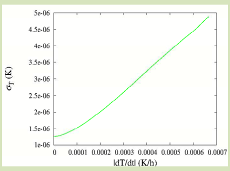
Parameter estimation

The model error η_i is estimated as time series of “1h forecast – analysis” differences, forecasts are run every hour (Δt = 1h) during one month (January and July)

Then for each bin of a predictor (flow dependence!)

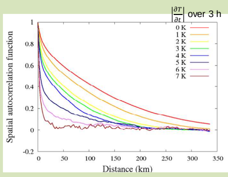
$$\gamma = \frac{1}{N}\sum_{i=1}^{N} \frac{\eta_{i+1} - \eta_i}{\eta_i \Delta t} \qquad \sigma = \frac{1}{N}\sum_{i=1}^{N} \frac{(\eta_{i+1} - \eta_i)^2}{\Delta t}$$





λ is determined from the empirically determined spatial autocorrelation function $G(\vec{r})$ through the solution of the implicit equation wrt λ for a particular spatial lag $\vec{r}$ (Garcia-Ojalvo at al., 1992)

$$G(\vec{r}) = \sum_{\vec{k}} \frac{\cos(\vec{k}\cdot\vec{r})}{1+\lambda^2 \vec{k}^2}$$



General considerations

Stochastic approach: run an ensemble of forecasts with a randomly simulated model error.

The spread between ensemble members serves as an estimate of the model uncertainty.

The strategy to simulate the model error is

- to approximate the empirically determined error of the model tendencies by a random process with the same statistical properties;
- to add this estimate of the model tendency error to the right-hand side of the governing equations.

Disadvantage: lack of understanding of essential physics of model error

Advantages: the entire model error is represented (important for data assimilation);

properties of the simulated model error, such as noise amplitude and time and space correlations, are not arbitrary

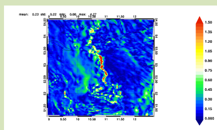
Present scheme vs. SPPT

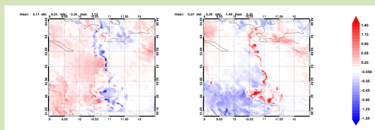
SPPT		present scheme
	predictors	
tendencies of the same variables		may be any state variable (determined by the analysis of the model error behaviour)
	parameters	
prescribed		flow-dependent, derived from the actual model error behaviour
	reaction	
instantaneous (the noise term is proportional to the tendency)		the noise is processed by the equation (supported by the analysis of the model error: in the COSMO model, the model error is not correlated with the tendency, but σ is)

First results

Simulation of the temperature error along a front, COSMO-DE, 01.01.2014, 00UTC + 3h

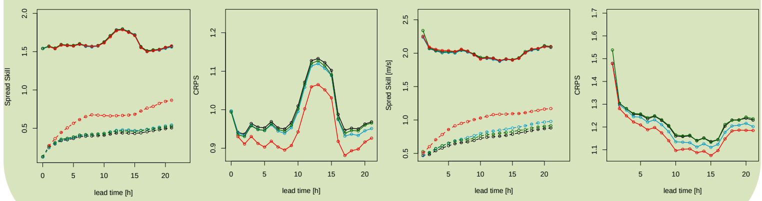
ABS(T_2m error) ens members – ens mean





COSMO-DE-EPS scores over a month, January 2014

T_2m Wind gusts 10 m



Control Random Parameters SPPT present scheme

Conclusions and outlook

A stochastic model for the model error is proposed, which is based on differential stochastic equations describing the flow-dependent time evolution of the error of the tendencies of model variables.
The parameters in the stochastic equations are dependent on resolved model variables, and this dependence is determined using an inexpensive training period.
In this way, the resulting perturbations represent the behaviour of the model error appropriate to the current modelled weather conditions.
Initial tests are carried out to assess the performance of the scheme in the full operational setup, and the first results show that the scheme is able to provide an increase in spread which corresponds well with the model error.

J. Garcia-Ojalvo et al., Generation of spatiotemporal colored noise, *Phys. Rev. A*,1992, V. 46, N.8, pp. 4670-4675