TECHNICAL MEMORANDUM

261

Influence of physical processes on the tangent linear approximation

J-F. Mahfouf

Research Department

October 1998 (accepted for publication in Tellus)

European Centre for Medium-Range Weather Forecasts
Europäisches Zentrum für mittelfristige Wettervorhersage
Centre européen pour les prévisions météorologiques à moyen

Abstract

A comprehensive set of linear physical parametrizations is developed for the tangent-linear and adjoint versions of ECMWF global forecast model. The following processes are described : vertical diffusion, subgrid-scale orographic effects, large scale condensation, longwave radiation and deep cumulus convection. The accuracy of a tangent-linear model including these processes is examined by comparing 24-hour forecasts obtained from an adiabatic tangent-linear version and pairs of non-linear integrations including full physics. It is shown that for finite size perturbations (analysis increments), the inclusion of physics improves the fit to the non-linear model. The improvement is largest for specific humidity, where combined effects of vertical diffusion (near the surface), large-scale condensation (in the mid-latitudes troposphere) and moist convection (in the lower tropical troposphere) contribute to a better evolution of moisture increments. Simplications have been designed with respect to the operational non-linear physics, mostly to avoid the growth of spurious unstable modes. Computation of singular vectors has revealed that the linear package does not contain such spurious unphysical structures. This comprehensive set of linear parametrizations is currently used in the ECMWF operational 4D-Var assimilation system.

1 Introduction

The hydrodynamical equations governing atmospheric flows are fundamentally non linear and therefore can only be solved numerically. However, for particular basic flows, the linearized equations can provide a useful description of wave propagation and of unstable regimes of the atmosphere. Adiabatic tangent-linear versions of primitive equation models have recently been developed, where the linearization is performed around an evolving trajectory. The tangent-linear models and their adjoints are used in variational data assimilation for an efficient computation of the gradient of a cost-function with respect to initial conditions (Thépaut and Courtier, 1991 ; Rabier et al., 1997). They are also used to compute the fastest growing modes (singular vectors) in order to span the phase space for ensemble prediction systems (Buizza, 1994 ; Molteni et al., 1996). The success of these applications lies in the ability of adiabatic tangent-linear models to accurately describe the development stage of baroclinic waves in mid-latitudes.

Diabatic processes are known to be important near the surface through momentum dissipation and in the free atmosphere through latent heat release. In particular, the distribution of diabatic heating produced by convection and radiation is fundamental to our knowledge of the tropical circulation.

The best way to include physical processes in tangent-linear models is still a matter of debate, since these processes are strongly non-linear and generally involve many conditionals. Recent results show that 4D-Var assimilation with physics allows the use of new types of observations, like precipitation data (Zupanski and Mesinger, 1995 ; Zou, 1996; Tsuyuki, 1996a,b). For highly simplified models, Xu (1996) has defined a generalized adjoint formalism which accounts for on/off switches. However, its utility for operational developments remains unclear. The accuracy of the linearization of a mesoscale model including moist physics has been examined by Vukicevic and Errico (1993), and more recently by Errico and Raeder (1998). They demonstrate that the accuracy of linearized moist physics is case dependent, but is generally better for atmospheric situations that are driven by the dynamics rather than by convective processes.

In this study, a comprehensive set of physical parametrizations is developed for the linearized versions of the ECMWF global model with applications in data assimilation, singular-vector calculations and sensitivity studies. In Section 2, simplifications of the linearized physics with respect to the operational physics are explained. After a description of the linearized parametrizations in Section 3, time evolutions of finite perturbations produced by the tangent-linear model are compared in Section 4 with their counterparts generated by the difference between two non-linear integrations. Emphasis is put on potential problems that may result if the linearization does not exclude the possibility of spurious growth mechanisms in the physical processes. The results are summarized in Section 5.

2 Methodology

Many studies with diabatic tangent-linear models have shown that the inclusion of physics reduces the range of validity of the tangent-linear approximation, and that the presence of thresholds in physical parametrizations can affect the convergence of the minimization process (Verlinde and Cotton, 1993 ; Zupanski, 1993). For important practical applications, linearized versions of forecast models are run at lower resolution than non-linear models. Examples include the incremental approach of the four-dimensional variational assimilation (4D-Var) as described in Courtier et al. (1994), and the initial perturbations computed for ensemble prediction systems (Mureau et al., 1996). This has direct consequences on the way physical processes should be included in linear models. Incremental 4D-Var uses a high resolution non-linear model with full physics to compute the fit to observations and a low resolution linear model with simplified physics to solve the minimization problem in terms of increments. The procedure guaranties a reasonable consistency between the low-resolution minimization and the actual high-resolution problem. The incremental approach also allows for a progressive inclusion of physical processes in the low resolution models with possible simplifications. Indeed, physical parametrizations can already behave differently between non-linear (NL) and tangent-linear (TL) models due to the change of resolution.

As underlined by Errico and Reader (1998), for practical applications the accuracy of the tangent-linear approximation should be checked for finite perturbations comparable in magnitude to the typical size of errors in our knowledge of the atmospheric state. The tangent-linear approximation of physical processes must also be examined with infinitesimal perturbations, but the purpose of such evaluation is restricted to the validation of numerical codes generated. For small perturbations, dealing with conditionals in a classical way (where the choice is provided by the basic state), there is almost always a regime where the linearization is very accurate. Given the fact that physical parametrizations are non-linear with thresholds, a linearized physical process will rarely provide the same response for finite size perturbations as the difference between two non-linear runs. Therefore, an important question arises : Do linearized physical parametrizations contain useful information for practical problems ? For example, could an approximate gradient be sufficient to solve the minimization problem even if the convergence is slowed down ?

Previous studies on linearized physical parametrizations were mostly focused on problems arising from the presence of thresholds, which may not be as critical in the incremental framework. In our view, a non-linear model with full physics should be considered as the appropriate standard against which to evaluate versions of tangent-linear models. Indeed, we shall demonstrate that the inclusion of physical processes can actually improve the accuracy of tangent-linear models. However, the mismatch between tangent-linear and non-linear evolutions can be very large even for differentiable parametrizations. Strong non-linearities can lead to erroneous behaviour of tangent-linear models. Problems created by spurious instabilities induced by positive feedback loops can be as important as those induced by thresholds in the physical parametrizations. They will be described and explained in Section 4.2.

3 Description of the model

The model considered in this study is a low resolution version of the ECMWF global spectral model with 31 vertical levels. The chosen triangular truncation at total wavenumber 42 is also used for singular vector computations and for a simplified Kalman filter. Sensitivity studies (Rabier et al., 1996) and the inner loop of 4D-Var (Rabier et al., 1997) are currently performed with a slightly higher resolution (T63) but still much lower than the truncation used for deterministic forecasts (T213). Both non-linear and tangent-linear versions are run using an Eulerian advection scheme with a 20 minute time step. The integrations are performed over a 24-hour period which is intermediate between the optimization time of singular vectors (48 h) and of current operational 4D-Var assimilation (6 h).

3.1 The non-linear physics

The physical processes included in the non-linear model are the same as in the operational ECMWF model (Cycle 16r2) except for the cloud scheme. They are summarized as follows :

- A vertical diffusion scheme based on a non-local K-type approach for convective boundary layers, and on exchange coefficients depending upon the local Monin-Obukhov length in stable conditions (Beljaars, 1995).
- A shortwave radiation scheme with two spectral intervals based on the two-stream approximation, and a longwave radiation scheme with six interval bands based on the emissivity approach (Morcrette, 1990).
- A representation of subgrid-scale orographic effects including low-level blocking and gravity wave breaking (Lott and Miller, 1997).
- A large scale condensation scheme described by a local moist adjustment process, including snowfall melting and rainfall evaporation.
- A diagnostic cloud scheme for stratiform and convective cloudiness according to Slingo (1987).
- A mass-flux convection scheme representing shallow, deep and mid-level convection types (Tiedtke, 1989).
- A land surface scheme with four soil layers accounting for the effects of a vegetation canopy (Viterbo and Beljaars, 1995).

3.2 The linearized physics

The linearized physics presented here describes five of the above processes. In order to prevent spurious unstable perturbations from growing, a number of simplifications have been defined. They will be justified more thoroughly in section 4.2. For the sake of simplicity, all the equations presented in this section do not consider pressure perturbations although they have been actually coded in the tangent-linear schemes.

3.2.1 Vertical diffusion

A vertical diffusion scheme based on a K-type closure with exchange coefficients depending upon the local Richarsdon number Ri as described in Louis et al. (1982) is adopted. Analytical expressions are generalized to the situation of different roughness lengths for heat and momentum transfers. The mixing length profile $l(z)$ uses the formulation of Blackadar (1962) with a reduction in the free atmosphere. For any conservative variable ψ (wind components u, v; dry static energy s; specific humidity q), the tendency of its perturbation ψ' produced by vertical diffusion is :

$$\frac{\partial \psi'}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(K(Ri)\frac{\partial \psi'}{\partial z}\right) \tag{1}$$

where ρ is the air density.

The exchange coefficient K is given by the following expression :

$$K = l^2 \|\frac{\partial \vec{V}}{\partial z}\| f(Ri) \tag{2}$$

In Equation (1), perturbations of the exchange coefficients have been neglected $K' = 0$. An heuristic example will be given in the following to justify this approximation.

3.2.2 Subgrid-scale orographic effects

The low level blocking part of the operational scheme developed by Lott and Miller (1997) is represented in the linear scheme. The tendency for wind perturbations $\vec{V}'$ produced by the low level drag processes is :

$$\frac{\partial \vec{V}'}{\partial t} = \frac{1}{\rho}\frac{\partial \vec{\tau}'}{\partial z} \tag{3}$$

where $\vec{\tau}'$ is the perturbation in momentum flux.

3.2.3 Large-scale condensation

We use a linearized version of a local moist adjustment scheme designed to produce stratiform precipitation.

If (q, T) is an initial supersaturated state $(q > q_s(T))$, a final adjusted state (q_*, T_*) such that $q_* = q_s(T_*)$ is reached by conserving moist static energy in a given atmospheric layer. The adjusted values are obtained through an iterative method with two updates.

The equations for the non-linear scheme are :

$$\begin{aligned} T_* - T &= -\lambda(q_s(T_*) - q) && (4) \\ q_* - q &= q_s(T_*) - q && (5) \end{aligned}$$

with $\lambda = L/C_p$, where L is the latent heat of liquid water and C_p is the specific heat at constant pressure. They are solved using a linearization of the specific humidity at saturation : $q_s(T_*) = q_s(T) + \gamma(T_* - T)$ with $\gamma = \partial q_s/\partial T$. Then, the linearization of $q_s(T')$ as $\gamma \times T'$, provides the equations for the tangent-linear scheme :

$$\begin{aligned} T'_* - T' &= \frac{-\lambda}{1+\lambda\gamma}(\gamma \times T' - q') && (6) \\ q'_* - q' &= \frac{1}{1+\lambda\gamma}(\gamma \times T' - q') && (7) \end{aligned}$$

Tendencies produced by large-scale condensation are thus :

$$\frac{\partial q'}{\partial t} = -\frac{1}{\lambda}\frac{\partial T'}{\partial t} = \frac{q'_* - q'}{2\Delta t} \qquad (8)$$

where Δt is the model time step.

This scheme also accounts for melting of snow whenever the temperature of a given layer exceeds 2 °C. Evaporation of precipitation in subsaturated layers is strongly reduced with respect to the non-linear scheme, as explained in Section 4.2.

This simple diagnostic adjustment scheme defers the problems of dealing with the complexity of the current ECMWF prognostic cloud scheme (Tiedtke, 1993).

3.2.4 Deep convection

The physical tendencies produced by convection on any conservative variable ψ can be written in a mass-flux formulation as (e.g. Betts, 1997):

$$\frac{\partial \psi}{\partial t} = \frac{1}{\rho}\left[(M_u + M_d)\frac{\partial \psi}{\partial z} + D_u(\psi_u - \psi) + D_d(\psi_d - \psi)\right] \qquad (9)$$

The first term on the right hand side represents the compensating subsidence induced by cumulus convection on the environment through the mass-flux M. The other terms account for the detrainment of cloud properties in the environment with a detrainment rate D. The subscripts u and d refer to the updrafts and downdrafts properties respectively. Evaporation of cloud water and precipitation should also be added in Equation (9) for s and q variables. Mass flux theory predicts that the effect of convection on large-scale temperature and moisture structures is dominated by compensating subsidence, as shown in the study of Gregory and Miller (1989). Therefore, non-linear tendencies produced by convection can be approximated by :

$$\frac{\partial \psi}{\partial t} \simeq \frac{1}{\rho}\left[(M_u + M_d)\frac{\partial \psi}{\partial z}\right] \tag{10}$$

This approximation is better for the heat budget than for the moisture budget (Betts, 1997). As a consequence, a partial linearization of Equation (10) is performed, where the vertical transport by the mean mass-flux is applied to the perturbations. From a non-linear integration of the convection scheme mass-flux profiles $M(z)$ are used to estimate the vertical transport of perturbed fields ψ' :

$$\frac{\partial \psi'}{\partial t} = \frac{1}{\rho}\left[(M_u + M_d)\frac{\partial \psi'}{\partial z}\right] \tag{11}$$

This linearized equation only applies to deep convection. The underlying assumption of Equation (11) is that perturbations of cloud properties can be neglected. The justification for such linear convection scheme is based on previous use of simplified physics at ECMWF. The experience with vertical diffusion for singular vectors (Buizza, 1994) has demonstrated that simplified physical parametrizations can improve the realism of a tangent-linear model, even if only part of the processes are described. The linear scheme developed by Buizza (1994) accounts only for neutral stability effects and uses a constant depth for the boundary layer, but is efficient enough to produce realistic momentum dissipation near the surface. Preliminary 4D-Var experiments by Rabier et al. (1998) have also demonstrated that it is possible to improve the analysis of specific humidity by relaxing, in the linearized models, the mean vertical velocity towards zero in the tropical belt. This encouraging result indicates that it is possible to describe the effect of cumulus induced subsidence in a more physical way with potential impact on 4D-Var. The present simplified linearized convection scheme must be seen as a first step before handling the complexity of the full operational ECMWF scheme. Results described in Section 4.1 will prove a posteriori that such simple scheme can improve significantly the time evolution of analysis increments in tropical regions.

3.2.5 Longwave radiation

Radiative transfer for the longwave spectrum is modeled using a constant emissivity formulation which was previously developed in the operational ECMWF forecast model to reduce the cost of radiative computations. This approach requires the storage of effective emissivity arrays ε computed from the full non-linear radiation scheme. Effective emissivity is defined in terms of the irradiance F and the local temperature T :

$$\varepsilon = \frac{F}{\sigma T^4} \tag{12}$$

The tendency of the perturbed temperature T' can thus be written:

$$\frac{\partial T'}{\partial t} = -\alpha \frac{g}{C_p} \frac{\partial}{\partial p}(4\varepsilon\sigma T^3 T') \tag{13}$$

with :

$$\alpha = \frac{1}{1 + \left(\frac{p_r}{p}\right)^{10}} \tag{14}$$

in order to reduce the radiative tendencies above $p_r = 300$ hPa (approximately in the stratosphere). Equation (13) implies that the net longwave radiation at any level strongly depends on the temperature at that level. This is only a reasonable approximation for optically thick atmospheres. In the stratosphere, as already stated by Thuburn (1994), the net longwave irradiance depends most strongly on conditions far below in the troposphere. It will be shown in section 4.2 that the effective emissivity method is unstable in the stratosphere. Therefore the use of the linear longwave scheme is restricted to the troposphere. A proper consideration of cloud-radiation interactions will require the development of a simple radiation scheme that could be called at every model time step (instead of every three hours as in the current operational model).

4 Tests of the accuracy of the tangent linear physics

We use two reference 24-hour non-linear forecasts starting from 7 Feb 1997 at 12 UTC produced by a T42L31 version of the ECMWF global spectral model : one starting from a background field $\mathbf{x_b}$ (6-hour forecast) and the other one starting from an analysis field $\mathbf{x_a}$. The difference between these two non-linear integrations is the standard reference of a number of tangent-linear integrations which propagate in time the analysis increments $\delta\mathbf{x} = \mathbf{x_a} - \mathbf{x_b}$, with the trajectory taken from the background field. The typical size for the analysis increments in the lower troposphere is about 3 m/s for wind speed, 1 K for temperature and 1 g/kg for specific humidity. This comparison should provide an upper bound for the validity of the tangent-linear approximation as the accuracy is improved when the size of the increments is reduced. In all the experiments, perturbations of the surface temperature and soil moisture variables are not considered. Surface variables from the trajectory are stored in order to provide a correct surface energy budget for the basic state. Since physical processes are integrated temporally with either forward or backward schemes, the prognostic variables from the trajectory at time $(t - \Delta t)$ need to be stored at each model time step. Since the ECMWF physics uses the method of fractional time steps (Beljaars, 1991) an additional storage of the adiabatic tendencies is necessary.

4.1 Results

4.1.1 Comparison of mid-latitudes and tropics

Most comparisons between TL and NL models have been performed with limited area models in mid-latitudes (Zupanski, 1993 ; Vukicevic and Errico, 1993). Global linear models have also been developed with moist physics, but the accuracy of the tangent-linear approximation has not been evaluated in the tropical belt where moist convection should play a dominant role for improving 4D-Var assimilation (Zou et al. 1993 ; Tsuyuki, 1996a,b).

To assess the importance of non-linearities produced by physical processes two sets of non-linear integrations are compared : one with a perturbation $(\mathbf{x_a} - \mathbf{x_b})$ added to the background field ("plus run"), and the other with a perturbation of opposite sign : $(\mathbf{x_b} - \mathbf{x_a})$ ("minus run"). Then, these two non-linear integrations are also compared to a tangent-linear integration. Both non-linear and tangent-linear models are run either without physics (adiabatic) or with full physics as described in Section 3. Two study regions are chosen : Western Europe which is representative of mid-latitudes and the Equatorial Atlantic which is representative of the tropics.

24-hour forecasts of analysis increments for temperature at level 27 (around 900 hPa) over Western Europe are presented in Figures 1 and 2 for adiabatic and diabatic evolutions respectively. The corresponding forecasts over Equatorial Atlantic are illustrated in Figures 4 and 5. Over Western Europe, there is a good level of agreement between the two non-linear runs and also with the tangent-linear evolution. However, even with an adiabatic model some differences are noticable (Figure 1). In the mid-Atlantic (50N,40W), the dipole of warm and cold anomalies differs significantly between the three integrations. In this region, the structure of perturbations produced by the TL model cannot match both non-linear behaviours. The TL negative anomaly is closer to the "minus run" whereas the TL positive anomaly is warmer by 0.25 K.

The NL evolutions with physics show further differences (Figure 2), such as a positive anomaly exceeding 2 K centered over Gibraltar in the "minus run" which has no counterpart in the "plus run" or in the TL run. However, some similarities between the three evolutions still exist, such as the dipole in the mid-Atlantic (50N,40W), and the positive maximum extending from the Channel over Central Europe. The positive anomaly off the coast of Portugal is also described by the three runs. In mid-latitudes, the physics generates some non-linearities, but the main features of the adiabatic response are kept. The dipole over the mid-Atlantic, which is associated to an area of intense precipitation, is enhanced, whereas the positive anomaly between France and Britain is weakened. The comparison between the two TL integrations shows that physical processes modifiy significantly the time evolution of analysis increments and lead to a better agreement with the NL integrations.

Since the physics influences both the trajectory and the increments, an additional experiment was performed to separate these two effects : the adiabatic TL model is linearized around a trajectory including full physics. The linearization is done that way at ECMWF for singular vector and sensitivity studies. Figure 3 shows that with a full physics trajectory, the adiabatic TL model is capable of increasing the intensity of the dipole over the Atlantic to a level comparable to the non linear runs. On the other hand, the positive anomaly over South Britain and Western France requires a TL model with physics to produce a response compatible with the NL model. This comparison demonstrates that the tangent-linear approximation is actually improved when the linearization is performed around a more realistic basic flow (non-linear evolution with full physics).

The tropical behaviour with adiabatic models shows comparable structures between the "plus" and "minus" NL runs (Figure 4). The inclusion of physical processes strongly modifies evolved increments (Figure 5). Most of the positive anomalies are strongly reduced (e.g. over the Gulf of Guinea) and the level of similarity between the two NL runs is also reduced (for example, there are no consistent anomalies larger than 0.5 K between the two NL runs). The difference between the NL and TL integrations is more important in the tropics than in mid-latitudes. However, the inclusion of physics improves the behaviour of the tangent-linear model by reducing the intensity of large positive anomalies simulated with the adiabatic model.

In summary, the influence of physical processes on the time evolution of analysis increments is quite marked in tropical regions. In mid-latitudes the physics modulates the intensity of the increments produced by the adiabatic model. The correlation between increments evolved with adiabatic models and increments evolved with diabatic models is lower in the tropics than in mid-latitudes. The benefical impact of the physics in the TL model is evident for both regions, although significant discrepancies with respect to the non-linear evolutions still persist in the tropics.

4.1.2 Zonal mean diagnostics

In this section the impact of the physics is described more quantitatively, and the relative importance of the various parametrization schemes is evaluated. For that purpose, mean absolute errors between NL and TL integrations are computed.

From two non-linear integrations of the forecast model M starting from differents conditions $\mathrm{x_a}$ and $\mathrm{x_b}$, the reference response is : $\Delta M = M(\mathrm{x_a}) - M(\mathrm{x_b})$. The response produced by a Version 1 of the tangent-linear model is : $\mathrm{M}_1(\mathrm{x_a}-\mathrm{x_b})$, and the response produced by a Version 2 is : $\mathrm{M}_2(\mathrm{x_a}-\mathrm{x_b})$. Let ϵ_i be the mean absolute error between NL and TL responses:

$$\epsilon_i = |\mathrm{M}_i - \Delta M| \tag{15}$$

The improvement produced by including more physics in Version 2 with respect to Version 1 should reduce the departure from the NL model, which implies $\epsilon_2 < \epsilon_1$. For example, let Version 1 be the adiabatic tangent-linear model and Version 2 the tangent-linear model with a

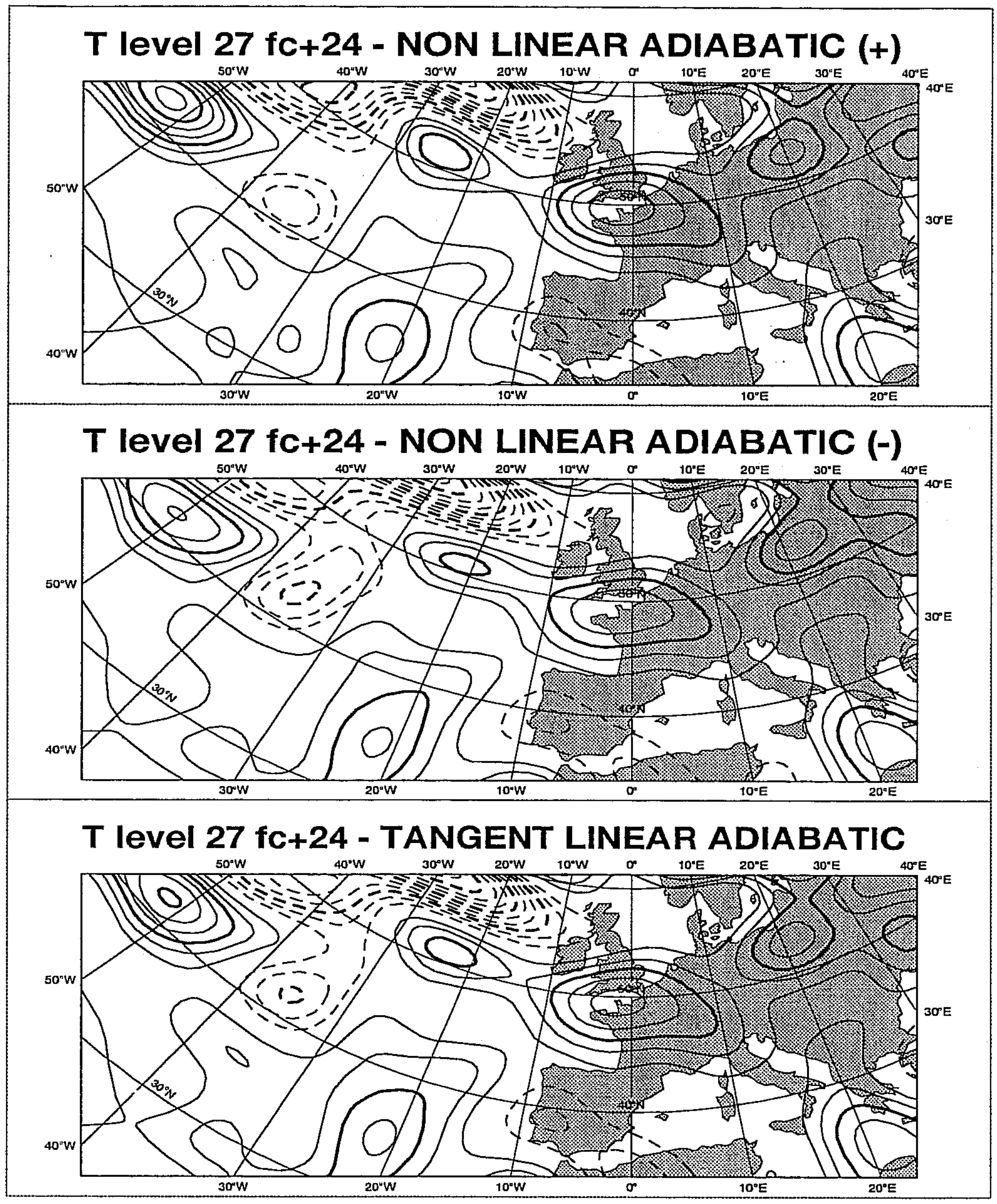

Figure 1: 24-hour evolution of temperature analysis increments at level 27 produced from the difference between two adiabatic non-linear integrations (upper and middle panels) and from an adiabatic tangent-linear integration (lower panel) over Western Europe. Initial perturbations of opposite signs are evolved in the two non-linear integrations. The opposite sign of the result is presented in the middle panel for a direct comparison with the upper panel. Contours are plotted every 0.25 K with negative values in dashed.

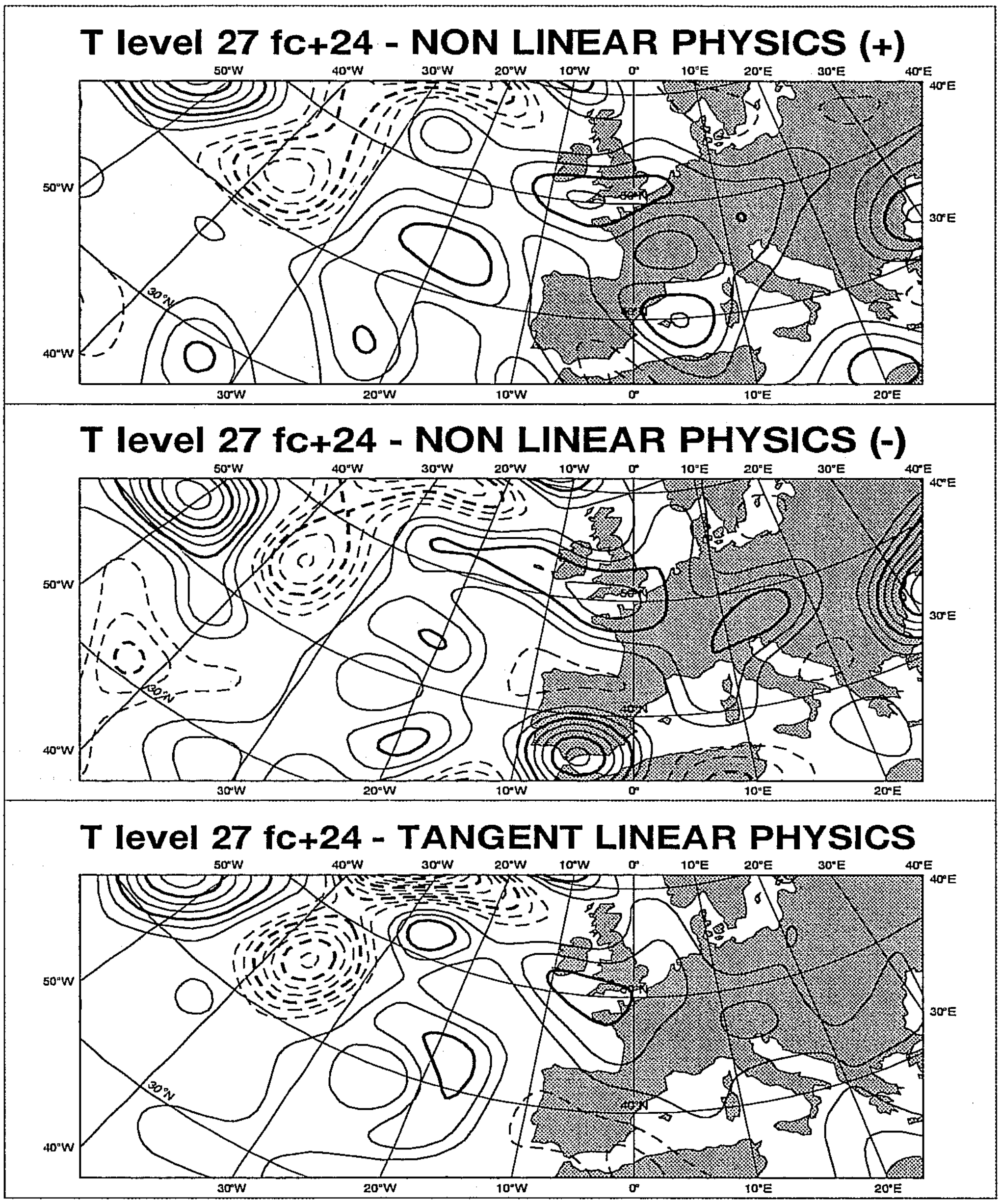

Figure 2: Same as Figure 1 but for non-linear and tangent-linear integrations including physical processes.

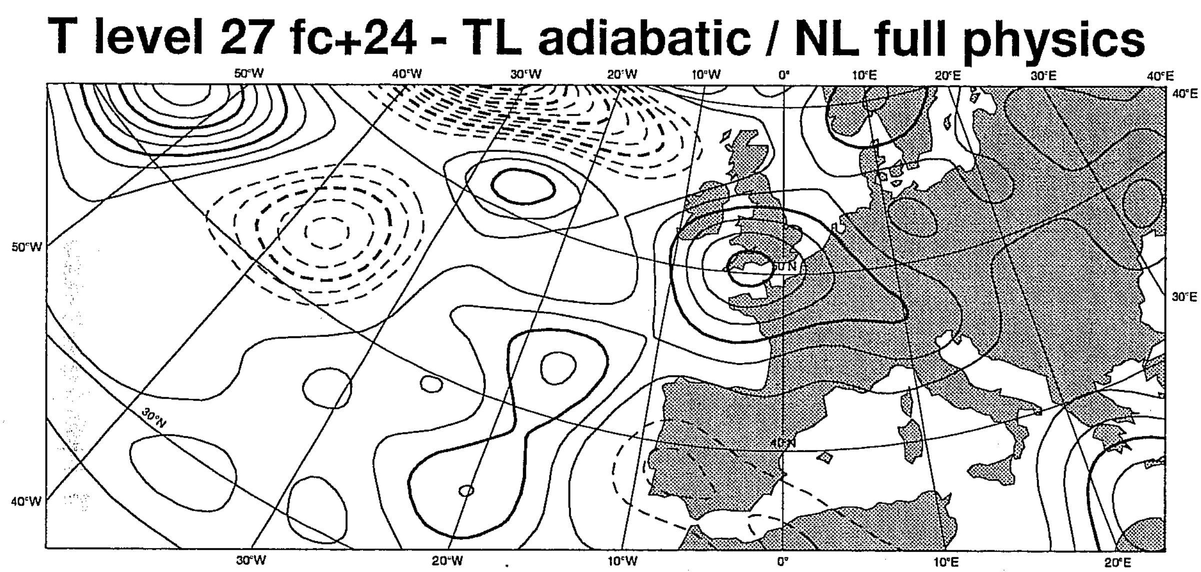

Figure 3: Same as Figure 1 (lower panel corresponding to the tangent-linear evolution) but where the increments are evolved with an adiabatic tangent-linear model around a trajectory computed with full physics.

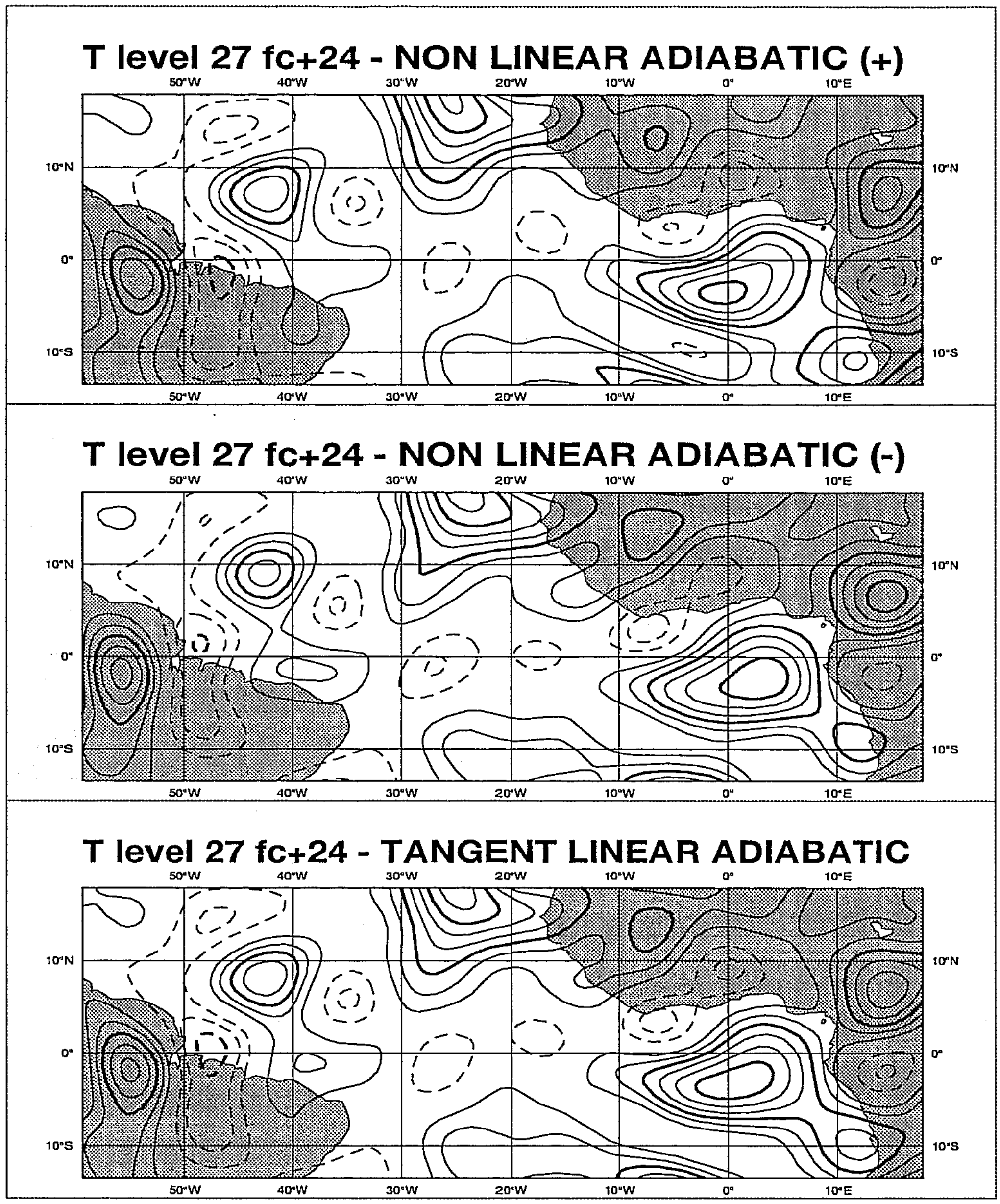

Figure 4: Same as Figure 1 but over the Tropical Atlantic.

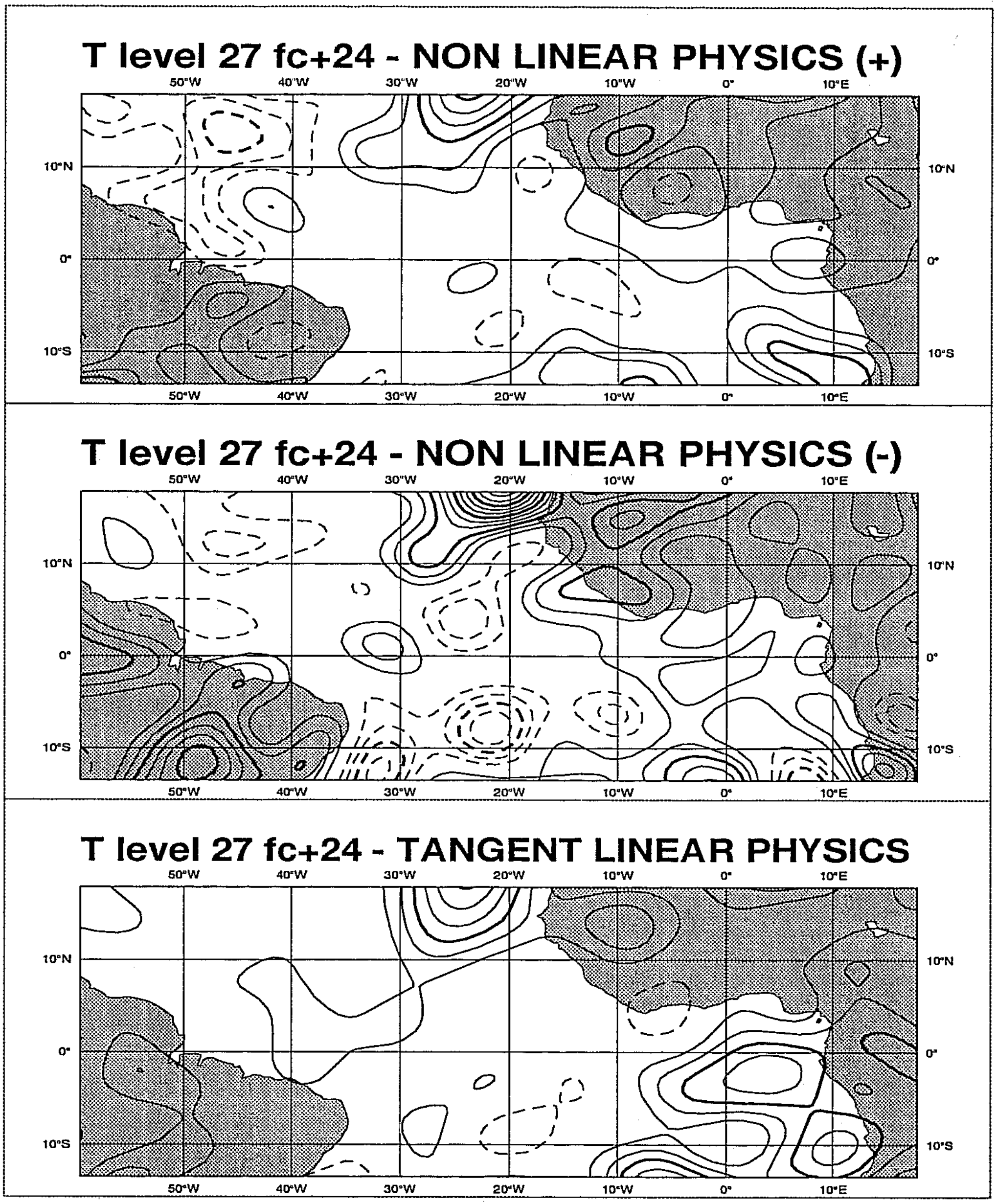

Figure 5: Same as Figure 2 but over the Tropical Atlantic.

given physical parametrization. If $\epsilon_2 > \epsilon_1$, the physical process makes the TL approximation worse than the adiabatic linearization, and therefore it should be either discarded or modified.

Figure 6a presents zonal mean errors for the zonal wind between the adiabatic TL model and the NL model with full physics. The global mean error is 0.59 m/s, and largest errors (above 1 m/s) are located close to the surface and in the upper tropical troposphere. Errors in the boundary layer come to the lack of surface friction. When introducing the vertical diffusion scheme, errors near the surface are significantly reduced leading to a global improvement of 9 % (Figure 6b). The improvement is maximum in the Northern Hemisphere where larger roughness lengths over continents enhance surface drag. The inclusion of the blocked flow part of the orographic drag also acts to reduce errors in zonal wind increments, below model level 25 (Figure 6c). The three main areas of error reductions are associated with high mountain ranges : Greenland, Himalaya, Rockies and the Antarctic plateau. The wave breaking part of the subgrid-scale orographic scheme was also tested, but the improvement is only marginal.

Figure 7a shows a similar picture to Figure 6a but for specific humidity. The mean absolute error is 0.14 g/kg for this field. The errors appear mostly in the tropical lower troposphere, where the largest values of the mean basic state are also located. Instead of a comparison with the adiabatic TL model, we have included all the dry processes (vertical diffusion, subgrid-scale orographic effects, radiation) in order to isolate the impact of moist physics. Due to strong interactions of moist convection with other physical processes, its influence can hardly be evaluated separately. The dry physics explains smaller errors in the boundary layer (vertical diffusion on specific humidity). The large-scale condensation (Figure 7b) reduces the global mean error by 8 %. The improvement is significant over the whole extra-tropical troposphere and in the higher tropical troposphere. On the other hand, in the lower equatorial troposphere error differences are positive which means that the inclusion of a large-scale condensation scheme is not beneficial. The lack of convection in this version of the TL model, increases the activity of large scale condensation in the tropics. Since the large scale condensation scheme only accounts for local processes, it cannot properly describe the vertical transport of moisture induced by convection. This shows the necessity of representing all the physical processes in a consistent way to produce a realistic evolution of analysis increments. The linearized deep convection scheme (Figure 7c) improves the fit to the NL model in the lower tropical troposphere where the impact of the large scale condensation was detrimental. These differences, although small (3 %) due to the non-linear behaviour of convection and to simplifications performed in the linearization, demonstrate the positive impact on moisture increments of the linear mass-flux scheme. It has been shown to be important for the analysis of specific humidity in the 4D-Var data assimilation system (Rabier et al., 1997). In order to more accurately evaluate the simplified mass-flux scheme, a similar experiment was performed with a complete linearization of a simpler convection scheme (Betts and Miller, 1993). The Betts-Miller scheme leads to a 5 % reduction of the mean error for specific humidity, which shows that the simplifications defined for the linearization of the mass-flux scheme are reasonable.

To conclude this part, the improved package of linear physics is compared to the simplified vertical diffusion scheme introduced by Buizza (1994) for the computation of singular vectors

TL version	u (m/s)	v (m/s)	q (g/kg)	T (K)
Simplified physics	0.54	0.52	0.15	0.28
Improved physics	0.49	0.47	0.12	0.26

Table 1: Mean absolute differences (in global mean) of two versions of the tangent-linear model for wind, specific humidity and temperature for a 24-hour forecast of analysis increments (7 Feb 1997). Simplified physics accounts for a vertical diffusion scheme with neutral stability. Improved physics accounts for the following processes : stability dependent vertical diffusion scheme, subgrid-scale orographic effects, large scale condensation, deep convection and longwave radiation

and for sensitivity studies. Table 1 presents the mean 24-hour errors with respect to non-linear integrations for both the simplified linear physics (neutral boundary layer with constant depth) and the improved linear physics. For wind speed and temperature, the mean error is reduced by 10 %, and the reduction reaches 20 % for specific humidity, because the simplified vertical diffusion scheme only applies to dry variables (wind and dry static energy).

4.2 Potential problems

In this section some specific problems on the linearization of physical parametrizations are addressed. The linear physics should represent the major feedback loops between the processes. However, positive feedbacks can lead to an exponential growth of initial perturbations, that the lack of non-linear effects will prevent from saturating with time. This means that when such behaviour takes place, the tangent-linear model can be useless. Various methods to identify and to solve these problems are illustrated in the following.

4.2.1 Longwave radiation

Figure 8 compares the impact of the linear radiation scheme on the tangent-linear approximation for the temperature field. Figure 8b describes the impact of the scheme presented in section 3.2.5, and Figure 8c the impact of the same scheme but applied everywhere (no damping in the stratosphere ; the parameter α in Equation (13) is set to one). In the troposphere, the positive impact is small because the short time scales (less than one day) associated with radiation through clouds interactions are not represented in the current linearization (no perturbations of the effective emissivity). A small positive impact is noticed near the surface where the constant emissivity approach is likely to be more valid due to a weaker contribution of remote layers to the temperature tendency. An interesting feature is noticed in the stratosphere where the inclusion of the linear longwave radiation scheme increases the mismatch with the NL model. The adiabatic TL model appears to be a better approximation of the full NL model.

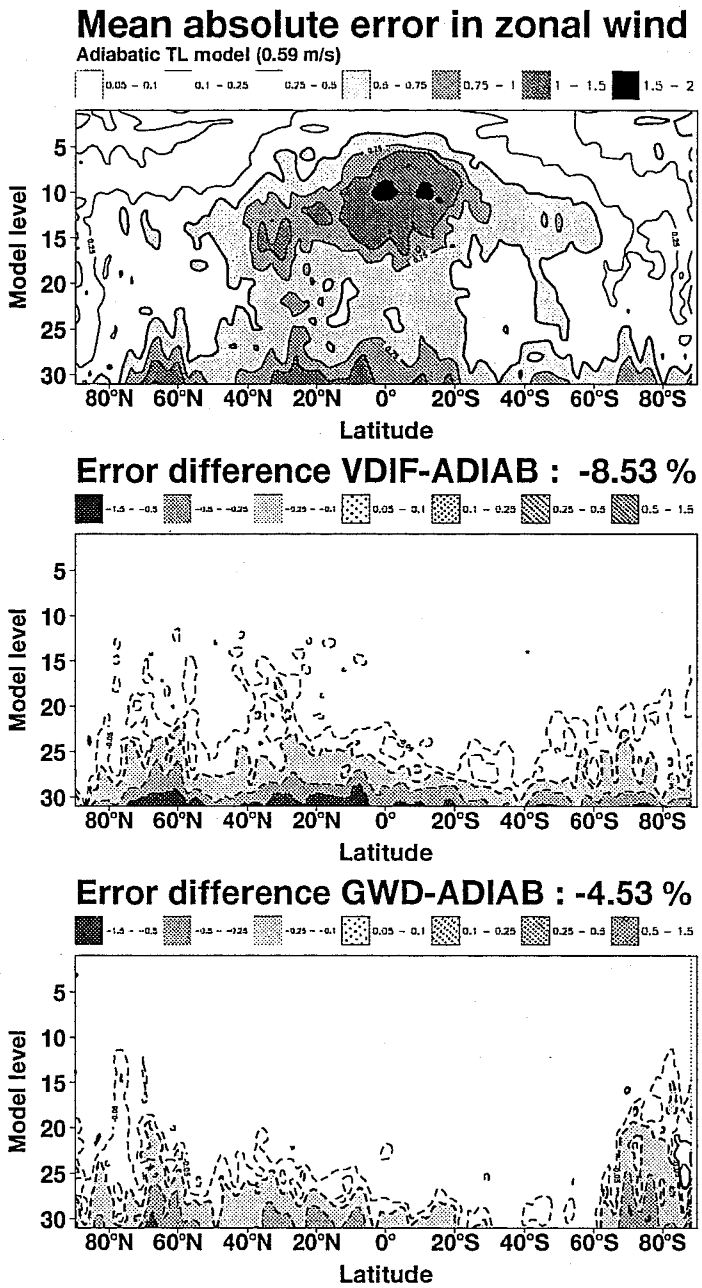

Figure 6: Influence of the tangent-linear vertical diffusion (VDIF) and subgrid-scale orography (GWD) parametrizations on the evolution of zonal wind increments (24-h forecast), compared with an adiabatic evolution (ADIAB). Results are presented in terms on mean absolute errors with respect to pairs of non-linear integrations with full physics (units : ms^{-1}). The upper panel (a) presents the differences between the adiabatic tangent-linear model and the non-linear model (difference between two integrations) with full physics. The middle panel (b) presents the error differences (in terms of fit to the non-linear model with full physics) between the tangent-linear model including vertical diffusion and an adiabatic tangent-linear model. The lower panel (c) presents the error differences between the tangent-linear model including subgrid-scale orographic effects and the adiabatic tangent-linear model.

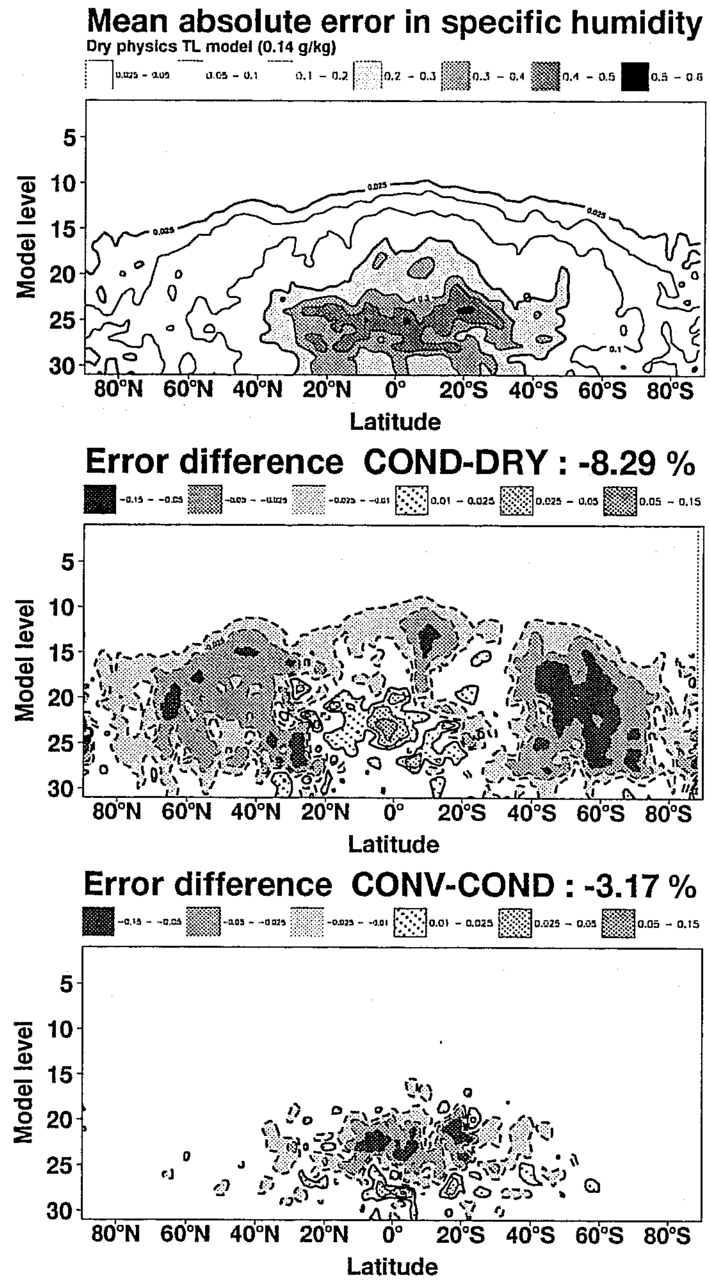

Figure 7: Influence of the tangent-linear moist processes [large-scale condensation (COND) and deep convection (CONV)] on the evolution of specific humidity increments (24-h forecast), compared to an evolution obtained with a linear model including only dry physical processes (DRY) (units : $g\ kg^{-1}$). The upper panel (a) presents the differences between the tangent-linear model with dry physics (vertical diffusion, longwave radiation, subgrid-scale orography) and the non-linear model with full physics. The middle panel (b) presents the error differences (in terms of fit to the non-linear model) between the tangent-linear model including large-scale condensation plus dry physics and the tangent-linear model with dry physics only. The lower panel (c) presents the error differences between the tangent-linear model with full physics and the tangent-linear model with full physics excluding deep convection.

By rewriting the evolution produced by radiation on the perturbed temperature field T' :

$$\frac{\partial T'}{\partial t} = -\frac{g}{C_p}\frac{\partial}{\partial p}(4\varepsilon\sigma T^3 T') \tag{16}$$

it can be splitted in two terms :

$$\frac{\partial T'}{\partial t} = -\frac{12g\varepsilon\sigma}{C_p}T^2\frac{\partial T}{\partial p} \times T' - \frac{4g\varepsilon\sigma}{C_p}\frac{\partial T'}{\partial p} \times T^3 \tag{17}$$

The first term on the right hand side of this equation can lead to an exponential growth of T' whenever the mean vertical temperature gradient $\partial T/\partial p$ is negative, which is the case in the stratosphere. However, the associated time constant is rather large (20 days). As a consequence, although it is better not to apply the linear radiation in the stratosphere, the spurious solution cannot propagate over the whole model during a 24-hour integration.

This problem was first identified by Thuburn (1994) when using a stratospheric version of the UGAMP general circulation model. It explains why a constant flux approach is now adopted in the operational ECMWF model for the spatial and temporal interpolations of the radiation scheme.

4.2.2 Vertical diffusion

The same type of problem can arise for the vertical diffusion, but with much shorter time scales. To illustrate these difficulties, the prognostic equation for surface temperature T_s is linearized assuming heat exchanges through turbulence only:

$$C_s\frac{\partial T_s}{\partial t} = \rho C_H|\vec{V}|(T_a - T_s) \tag{18}$$

where C_s is the heat capacity of the soil.

If perturbations of atmospheric parameters are neglected, then the tangent-linear equation writes:

$$C_s\frac{\partial T_s'}{\partial t} = -\rho|\vec{V}|\left[C_H + \frac{\partial C_H}{\partial T_s} \times (T_a - T_s)\right]T_s' \tag{19}$$

In stable conditions ($T_a - T_s > 0$), the heat transfer coefficient can be expressed as (Louis et al., 1982) :

$$C_H = \frac{C_{HN}}{(1 + \mu Ri^{3/2})} \tag{20}$$

where C_{HN} is the neutral transfer coefficient , μ an empirical coefficient and Ri the Richardson number.

As a consequence, $\partial C_H/\partial T_s$ can be written as:

$$\frac{\partial C_H}{\partial T_s} = \frac{\partial C_H}{\partial Ri}\frac{\partial Ri}{\partial T_s} = -\frac{3}{2}\mu C_{HN}\frac{\sqrt{Ri}}{(1+\mu Ri^{3/2})^2}\frac{Ri}{T_a - T_s} \quad (21)$$

By replacing this expression in Equation (19) one gets :

$$C_s\frac{\partial T_s'}{\partial t} = \rho C_{HN}|\vec{V}|\left[-1+\frac{3}{2}\mu\frac{Ri^{3/2}}{1+\mu Ri^{3/2}}\right]T_s' \quad (22)$$

When the factor in brackets is positive, corresponding to a Richardson number larger than $(2/\mu)^{2/3}$, an exponential growth of the surface temperature perturbation will take place. The occurence of such situations is more difficult to diagnose in the boundary layer due to the coupling between layers. Such non-physical unstable modes can be reduced in two ways. Either the tangent-linear model can be simplified by accounting only part of the linearization, that is :

$$C_s\frac{\partial T_s'}{\partial t} = -\rho C_H|\vec{V}|T_s' \quad (23)$$

which only represents the negative feedback term (when the surface temperature decreases, the downward heat flux increases). This approach, also chosen by Errico et al. (1993), has been adopted in this study by setting $K' = 0$ in Equation (1). On the other hand, the non-linear model can be modified to reduce the derivative $\partial C_H/\partial Ri$ in Equation (21). This smoothing approach has been retained by Janiskova et al. (1998). The main disadvantage of the second method lies in the subtle tuning of the smoothing parameters : they need to be small enough in order to provide similar results to the original non-linear scheme, but they must be large enough to prevent instabilities from growing in the tangent-linear scheme.

4.2.3 Singular vectors with moist physics

Problems mentionned above can be identified by computing singular vectors with a linear model including physical processes. Indeed, initial perturbations that grow fastest over one or two days are captured by this technique. Large growth rates, computed with the tangent-linear model, that cannot amplify in the non-linear model are associated with non-meteorological features (as shown by Buizza (1994) before the introduction of a simple vertical diffusion scheme). Preliminary computations of singular vectors with a linear mass-flux convection scheme based on the Jacobian approach have produced very highly unstable modes. Moreover, the energy spectrum of these singular vectors often has a maximum in the smallest scales resolved by the model.

The usefulness of singular vectors to identify non meteorological instabilities produced by the linearized physics is illustrated in details for the large-scale condensation scheme. Singular vectors are computed over a 48-hour optimization period starting from 7 Feb 1997 at 12 UTC with a T42L31 model (tangent-linear and adjoint) including the improved linear physics. In a second experiment, the large-scale condensation scheme is modified to represent evaporation of precipitation in subsaturated layers. The growth rate of initial perturbations is computed using an energy scalar product with a moisture term as described by Mahfouf et al. (1996), which allows non-zero perturbations of specific humidity at initial time. The vertical profile of total energy at final time for the first singular vector is presented on Figure 9. The effect of rainfall evaporation is to maximize total energy close to the surface and also at level 23, whereas the vertical structure without evaporation is similar to cross-sections presented by Buizza (1994) where the maximum of energy at final time is located in the upper troposphere. The initial perturbation leading to a large amplification close to the surface do not grow in the NL model. In consequence, it is better to remove the linearization of such process, since it can produce an evolution of initial perturbations uncompatible with their NL behaviour. This feature comes from a threshold handling in the linearized scheme. In a schematic way, the evaporation process is described by the following mechanism :

$$q = q + \Delta q \qquad \text{if} \quad q < q_s \tag{24}$$

where Δq represents the amount of evaporated rainfall. This quantity is proportional to the subsaturation $(q_s - q)$ in a given layer.

The growth of the mean specific humidity q is bounded by the saturation value q_s, but the same criterion also applies to the increments q', which means that as long as specific humidity from the basic state remains below saturation $(q < q_s)$, the perturbation can grow, provided perturbations $\Delta q'$ are large enough.

This problem should be investigated in more details in future studies. However, is is likely that these unstable modes are related to the use of the tangent-linear approximation for finite size perturbations and to the lack of feedbacks between the perturbation and the basic state. The last remark is specific to singular vector computations and should be less a problem in a 4D-Var incremental approach with several updates of the trajectory.

The spectrum of the first 12 singular vectors is compared in Figure 10 for the simplified vertical diffusion scheme of Buizza (1994) and for the improved physics presented in this paper. They look very similar, the inclusion of latent heat release in the linear model produces slightly larger growth rates through a decrease of static stability, in agreement with results from Ehrendorfer et al. (1998).

5 Conclusions and perspectives

In this paper, a comprehensive package of linear physics to be included in the tangent-linear and adjoint versions of the ECMWF forecast model for 4D-Var assimilation and singular-vector applications has been presented. A linearization of the main physical processes has been performed to improve the time propagation of analysis increments over a 24-hour period.

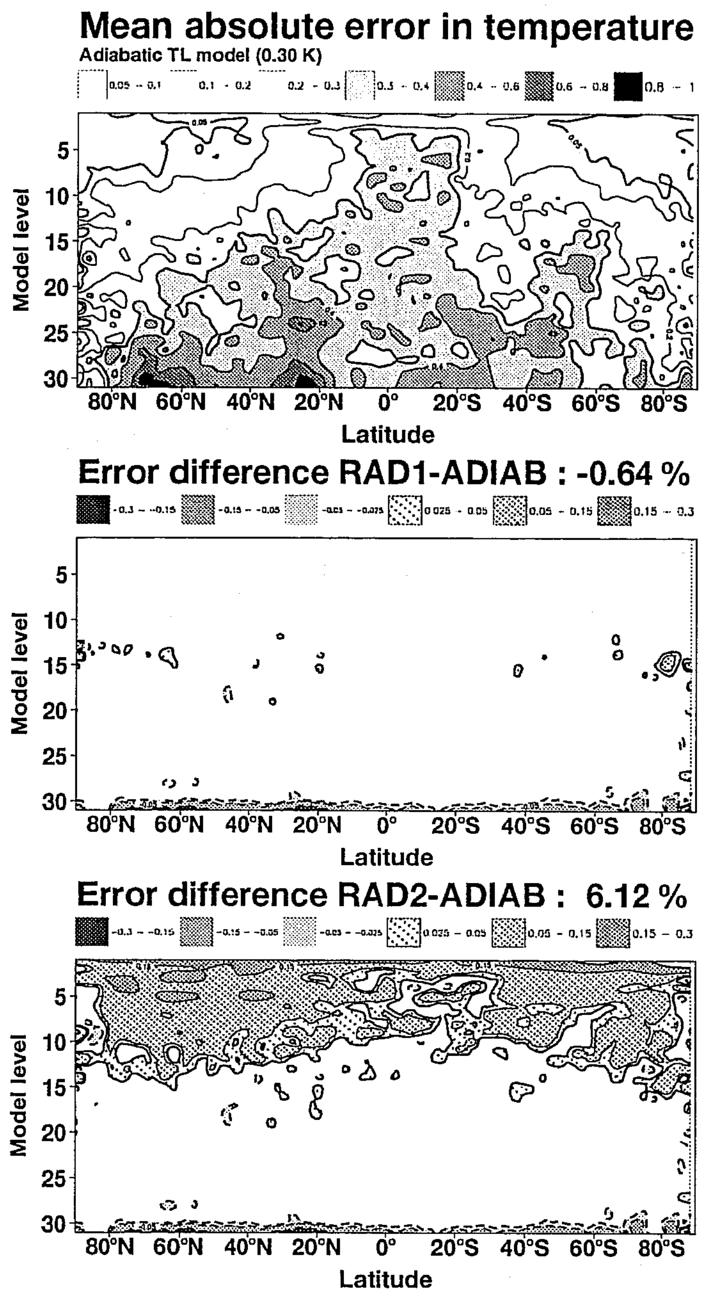

Figure 8: Influence of the tangent-linear longwave radiation in the stratosphere, on the evolution of temperature increments (24-h forecast). The adiabatic TL model (ADIAB) is compared to a version including longwave radiation in the troposphere only (RAD1) and everywhere (RAD2) (units : K). The upper panel (a) presents the differences between the adiabatic tangent-linear model and the non-linear model with full physics. The middle panel (b) presents the error differences (in terms of fit to the non-linear model) between the tangent-linear model with longwave radiation only in the troposphere and the adiabatic tangent-linear model. The lower panel (c) presents the error differences between the tangent-linear model with longwave radiation also in the stratosphere and the adiabatic tangent-linear model.

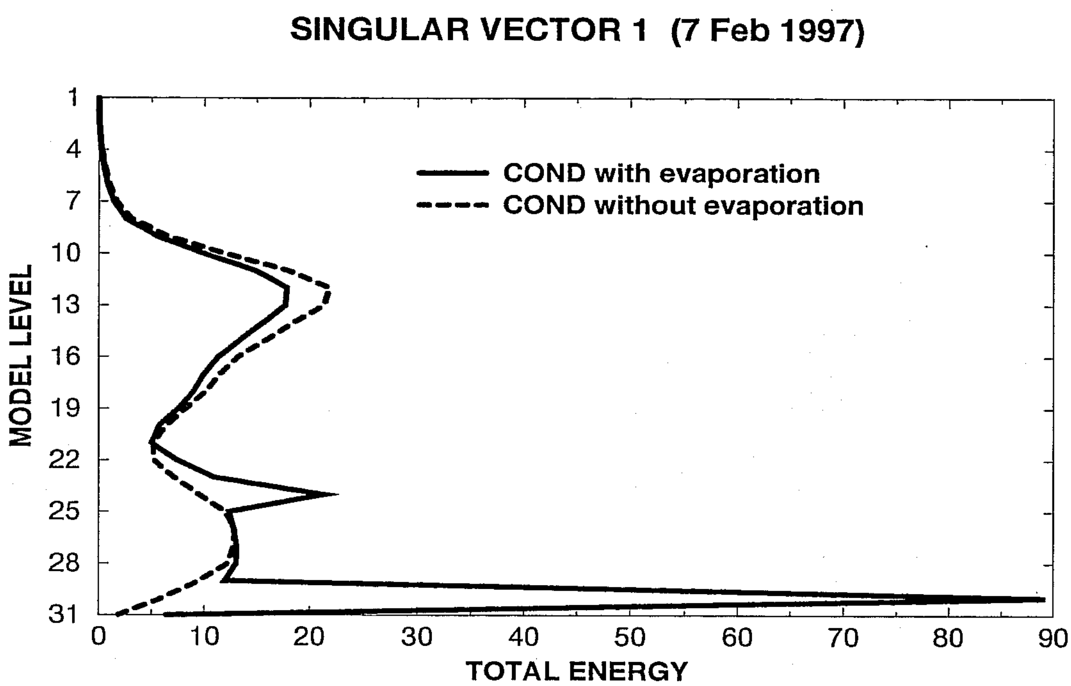

Figure 9: Influence of rainfall evaporation on the total energy profile of the first singular vector after a 48-hour tangent-linear evolütion starting on 7 Feb 1997. The solid line represents the vertical profile when evaporation of rainfall in taken into account in the linearized large-scale condensation parametrization, and the dashed line the vertical profile when evaporation of rainfall is strongly reduced.

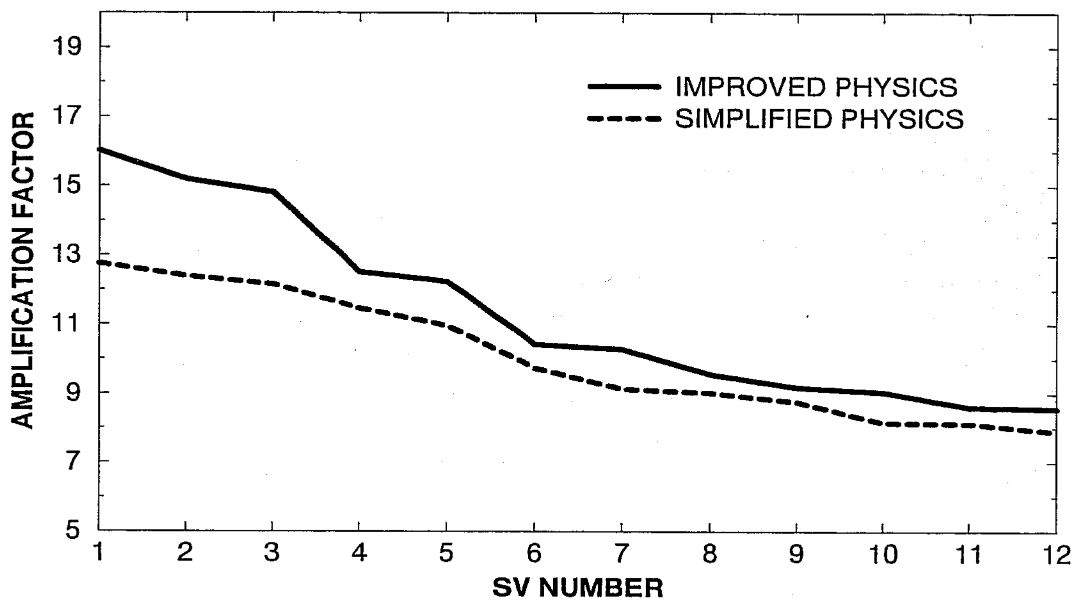

Figure 10: Spectrum of the first 12 singular vectors computed with a T42L31 linearized model including simplified physics (dashed line) and improved physics (solid line). Singular vectors are optimized over a 48-hour period above 30 ^{o}N.

All the physical processes linearized actually improve the fit to the non-linear model with full physics. Mean absolute errors are reduced by 20 % for specific humidity and by 10 % for other prognostic variables. Simplifications performed in the linearization have been justified to prevent the amplification of non-physical instabilities. The linear model with improved physics produces singular vectors in the extra-tropics consistent with those obtained with a simplified vertical diffusion scheme.

The next step of this preliminary work is to examine more precisely the influence of the linearized physics on singular vector calculation and 4D-Var assimilation. Of particular interest is the computation of moist singular vectors both for rapidly developing baroclinic systems and for the tropics. After an extensive evaluation (to be presented and analysed in forthcoming papers), this package of linear physics has been included within the first operational ECMWF 4D-Var data assimilation system. Results have shown a positive impact of the linear physics in the tropics, by reducing the spin-down in precipitation and improving wind scores (Rabier et al., 1997).

Improvements of the current linearized physics will concern the convection, where we plan to use the full adjoint of a comprehensive adjustement scheme (Betts and Miller, 1993) for precipitation rates retrievals, since very encouraging results have already been obtained by Fillion and Errico (1997) from a 1D-Var assimilation. In a one-dimensional framework, the behaviour of the mass-flux scheme can also be tested. Indeed, its complete adjoint can be obtained explicitly by computing the Jacobian elements from a perturbation method. This method is affordable for single-column assimilation, but would be too expensive for 4D-Var assimilation. The radiative transfer model also needs to be improved in order to account for cloud interactions, which is the dominant process given the time scale of interest (one day). The high computing cost of the operational ECMWF longwave code, requires alternative approaches to be explored such as those proposed by Chou and Neelin (1996) using Jacobians or by Chevallier et al. (1998) using neural networks. Before considering the complexity of the full prognostic cloud scheme which requires addressing the difficult problem of the specification of background error statistics for cloud variables, a diagnostic approach based on the simplification of the Tiedtke's scheme will be first developed.

Acknowledgements

I would like to thank M. Hamrud and L. Isaksen for their advice during the technical implementation of the linear physics in the tangent-linear and adjoint versions of the IFS model. I am also grateful to R. Errico for useful discussions during the early stages of the developments. Singular vectors with physics were developed in collaboration with R. Buizza. F. Rabier has been very helpful to design the post-processing within the tangent-linear model. Suggestions by A. Beljaars, F. Bouttier, A. Simmons, M. Miller and A. Hollingsworth greatly improved the first version of the paper.

References

Beljaars, A.C.M., 1991: Numerical schemes for parametrizations. In *ECMWF Seminar on Numerical methods in atmospheric models*, pages 1–42, Shinfield Park, Reading (UK). ECMWF. 9-13 September 1994.

Beljaars, A.C.M., 1995: The impact of some aspects of the boundary layer scheme in the ECMWF model. In *ECMWF Seminar on Parametrization of sub-grid scale physical processes*, pages 125–161, Shinfield Park, Reading (UK). ECMWF. 5-9 September 1994.

Betts, A.K., 1997: The parameterization of deep convection: a review. In *ECMWF Workshop on new insights and approaches to convective parametrization*, pages 166–188, Shinfield Park, Reading (UK). ECMWF. 4-7 November 1996.

Betts, A.K., and M.J. Miller, 1993: *The Betts-Miller scheme*, pages 107–121. American Meteorological Society, Eds. K.A. Emanuel and D.J. Raymond edition. Chapter 9 in *The Representation of cumulus convection in numerical models of the atmosphere.*

Blackadar, A.K., 1962: The vertical distribution of wind and turbulent exchange in a neutral atmosphere. *J. Geophys. Res.*, **67**, 3095–3102.

Buizza, R., 1994: Sensitivity of optimal unstable structures. *Quart. J. Roy. Meteor. Soc.*, **120**, 429–451.

Chevallier, F., F. Chéruy, N.A. Scott, and A. Chédin, 1998: A neural network approach for a fast and accurate computation of longwave radiative budget. *J. Appl. Meteor.*, **37**, 1385–1397.

Chou, C., and J.D. Neelin, 1996: Linearization of a longwave radiation scheme for intermediate tropical atmospheric model. *J. Geophys. Res.*, **101D**, 15129–15146.

Courtier, P., J.-N. Thépaut, and A. Hollingsworth, 1994: A strategy for operational implementation of 4D-Var using an incremental approach. *Quart. J. Roy. Meteor. Soc.*, **120**, 1367–1387.

Ehrendorfer, M., R.M. Errico, and K.D. Raeder, 1998: Singular-vector perturbation growth in a primitive-equation model with moist physics. *J. Atmos. Sci.*, (accepted for publication).

Errico, R.M., and K.D. Reader, 1998: An examination of the accuracy of the linearization of a mesoscale model with moist physics. *Quart. J. Roy. Meteor. Soc.*, (accepted for publication).

Errico, R.M., T. Vukicevic, and K. Reader, 1993: Examination of the accuracy of a tangent linear model. *Tellus*, **45A**, 462–477.

Fillion, L., and R. Errico, 1997: Variational assimilation of precipitation data using moist-convective parameterization schemes: A 1DVAR study. *Mon. Weather Rev.*, **125**, 2917–2942.

Gregory, D., and M. Miller, 1989: A numerical study of the parametrization of deep tropical convection. *Quart. J. R. Met. Soc.*, **115**, 1209–1241.

Janiskova, M., J.-N. Thépaut, and J.-F. Geleyn, 1998: Simplified and regular physical parametrizations for incremental four-dimensional variational assimilation. *Mon. Weather Rev.*, . (accepted for publication).

Lott, F., and M.J. Miller, 1997: A new subgrid-scale drag parametrization : Its formulation and testing. *Quart. J. Roy. Meteor. Soc.*, **123**, 101–127.

Louis, J.-F., M. Tiedtke, and J.-F. Geleyn, 1982: A short history of the operational PBL parametrization at ECMWF. In *ECMWF Workshop on boundary layer parametrization*, pages 59–79, Shinfield Park, Reading (UK). ECMWF. November 1981.

Mahfouf, J.-F., R. Buizza, and R.M. Errico, 1996: Strategy for including physical processes in the ECMWF variational data assimilation system. In *ECMWF Workshop on non-linear aspects of data assimilation*, pages 595–632, Shinfield Park, Reading (UK). ECMWF. 9-11 September 1996.

Molteni, F., R. Buizza, T.N. Palmer, and T. Petroliagis, 1996: The ECMWF Ensemble Prediction System : methodology and validation. *Quart. J. Roy. Meteor. Soc.*, **122**, 73–119.

Morcrette, J.-J., 1990: Impact of changes to the radiation transfer parametrization plus cloud optical properties in the ECMWF model. *Mon. Weather Rev.*, **118**, 847–873.

Rabier, F., E. Klinker, P. Courtier, and A. Hollingsworth, 1996: Sensitivity of forecast errors to initial conditions. *Quart. J. Roy. Meteor. Soc.*, **122**, 121–150.

Rabier, F., J.-F. Mahfouf, M. Fisher, H. Jarvinen, A. Simmons, E. Andersson, F. Bouttier, P. Courtier, M. Hamrud, J. Haseler, A. Hollingsworth, L. Isaksen, E. Klinker, S. Saarinen, C. Temperton, J.-N. Thépaut, P. Undén, and D. Vasiljević, 1997: Recent experimentation on 4D-Var and first results from a Simplified Kalman Filter. Technical Report 240, E.C.M.W.F., Shinfield Park, Reading , U.K. ECMWF Tech. Memo.

Rabier, F., J.-N. Thépaut, and P. Courtier, 1998: Extended assimilation and forecast experiments with a four-dimensional variational assimilation system. *Quart. J. Roy. Meteor. Soc.*, **124**, 1861–1887.

Slingo, J.M., 1987: The development and verifications of a cloud prediction scheme of the ECMWF model. *Quart. J. Roy. Meteor. Soc.*, **113**, 899–927.

Thépaut, J.-N., and P. Courtier, 1991: Four dimensional variational assimilation using the adjoint of a multilevel primitive equation model. *Quart. J. Roy. Meteor. Soc.*, **117**, 1225–1254.

Thuburn, J., 1994: Stratospheric modelling within UGAMP. In *ECMWF Workshop on stratosphere and numerical weather prediction*, pages 193–212, Shinfield Park, Reading (UK). ECMWF. 15-17 November 1993.

Tiedtke, M., 1989: A comprehensive mass-flux scheme for cumulus parameterization in large-scale models. *Mon. Weather Rev.*, **117**, 1779–1800.

Tiedtke, M., 1993: Representation of clouds in large-scale models. *Mon. Weather Rev.*, **121**, 3040–3061.

Tsuyuki, T., 1996a: Variational data assimilation in the tropics using precipitation data. Part I: Column Model. *Meteor. and Atmos. Phys.*, **60**, 87–104.

Tsuyuki, T., 1996b: Variational data assimilation in the tropics using precipitation data. Part II: 3D Model. *Mon. Weather Rev.*, **124**, 2545–2561.

Verlinde, J., and W.R. Cotton, 1993: Fitting microphysical observations of nonsteady convective clouds to a numerical model: An application of the adjoint technique of data assimilation to a kinematic model. *Mon. Weather Rev.*, **121**, 2776–2793.

Viterbo, P., and A.C.M. Beljaars, 1995: An improved land surface parametrization scheme in the ECMWF model and its validation. *J. Climate*, **8**, 2716–2748.

Vukicevic, T., and R.M. Errico, 1993: Linearization and adjoint of parameterized moist diabatic processes. *Tellus*, **45A**, 493–510.

Xu, Q., 1996: Generalized adjoint for physical processes with parameterized discontinuities. Part I: Basic issues and heuristic examples. *J. Atmos. Sci.*, **53**, 1123–1142.

Zou, X., 1996: Tangent-linear and adjoint of 'on-off' processes and their feasibility for use in 4-dimensional variational data assimilation. *Tellus*, **49A**, 3–31.

Zou, X., I.M. Navon, and J.G. Sela, 1993: Variational data assimilation with moist threshold processes using the NMC spectral model. *Tellus*, **45A**, 370–387.

Zupanski, D., 1993: The effects of discontinuities in the Betts-Miller cumulus convection scheme on four-dimensional variational data assimilation. *Tellus*, **45A**, 511–524.

Zupanski, D., and F. Mesinger, 1995: Four-dimensional variational assimilation of precipitation data. *Mon. Weather Rev.*, **123**, 1112–1127.