TECHNICAL MEMORANDUM

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Sea-surface roughness and drag coefficient as function of neutral wind speed

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July 2010

European Centre for Medium-Range Weather Forecasts
Europäisches Zentrum für mittelfristige Wettervorhersage
Centre européen pour les prévisions météorologiques à moyen terme

Series: ECMWF Technical Memoranda

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Abstract

Near the surface, it is commonly believed that the behaviour of the (turbulent) atmospheric flow can be well described by a constant stress layer. In the case of a neutrally stratified surface layer, this leads to the well-know logarithmic wind profile that determines the relation between near-surface wind speed and magnitude of stress. The profile is set by a surface roughness length, which, over the ocean surface, is not constant, but depends on the underlying (ocean-wave) sea state. At ECMWF, this relation is parametrized in terms of surface stress itself, where the scale is set by kinematic viscosity for light wind and a Charnock parameter for strong wind. For given wind speed at a given height, the determination of the relation between surface wind and stress (expressed by a drag coefficient), leads to an implicit equation that is to be solved in an iterative way.

In this document a fit is presented that directly expresses the neutral drag coefficient and surface roughness in terms of wind speed, without the need for iteration. Since the fit is formulated in purely dimensionless quantities, it is able to produce accurate results over a wide range for wind speed, level height and values for the Charnock parameter.

1 Introduction

In the ECMWF operational integrated forecast system (IFS), the lowest model level is currently designed to be close to a height of 10m. It is assumed that between this layer and the surface the constant (turbulent) stress assumption is valid, including a form of Monin-Obukhov stability theory. For a certain value of stress $\vec{\tau} = \rho_a u_* \vec{\mathbf{u}}_*$, where ρ_a is the air density, $\vec{\mathbf{u}}_*$ the friction velocity and u_* its magnitude, the following vertical equation is to be satisfied:

$$\frac{\partial \vec{\mathbf{u}}}{\partial z} = \frac{\vec{\mathbf{u}}_*}{\kappa(z+z_0)} \, \Phi_\mathrm{M}\left(\frac{z+z_0}{L}\right), \quad \vec{\mathbf{u}}(0) = 0. \tag{1}$$

Here $\kappa = 0.4$ is the Von Kármán constant, Φ_M is a stability-dependent gradient function and L is the Obukhov length. Detailed definitions on these quantities may be found in Part IV.3 of the IFS-documentation (2009). The strength of surface stress follows from integration of (1) and the details of the flow higher up in the atmosphere. The formal solution is given by:

$$\vec{\mathbf{u}}(z) = \frac{\vec{\mathbf{u}}_*}{\kappa}\left\{\log\left(\frac{z+z_0}{z_0}\right) - \Psi_\mathrm{M}\left(\frac{z+z_0}{L}\right) + \Psi_\mathrm{M}\left(\frac{z_0}{L}\right)\right\}, \tag{2}$$

where $\Phi_\mathrm{M}(\eta) = 1 - \eta\Psi'_\mathrm{M}(\eta)$.

Roughness length z_0 in this wind profile depends for light wind on the kinematic viscosity ν ($1.5x10^{-5}\mathrm{m^2s^{-1}}$) and on a Charnock relation (Charnock, 1955) for strong wind as:

$$z_0 = \alpha_\mathrm{M}\frac{\nu}{u_*} + \alpha_\mathrm{ch}\frac{u_*^2}{g}. \tag{3}$$

Here $\alpha_\mathrm{M} = 0.11$, $g = 9.81\mathrm{ms^{-2}}$ is the gravitational acceleration, and α_ch depends on the (ocean-wave) sea state (Janssen, 1991). The common range of the latter parameter is from 0.01 for swell up to 0.04 for steep young ocean waves; although values up to 0.1 do sporadically occur. A typical value is 0.018.

The Obukhov length L depends on the stratification of the surface layer, and its evaluation requires the simultaneous solution for similar equations for heat and moisture, as well as the knowledge of the sea-surface temperature. Over the global oceans its contribution to (2) is usually modest, and adds on average $0.2\mathrm{ms^{-1}}$ to the 10m wind speed (see *e.g.*Brown *et al.* (2006)).

The concept of equivalent neutral wind u_n is a popular quantity that expresses the relation between surface stress and wind, where stability effects are excluded:

$$u_n = \frac{u_*}{\kappa} b_n, \quad \text{with} \tag{4}$$

$$b_n = \log(1 + z/z_0). \tag{5}$$

Here quantity b_n is directly related to the neutral drag coefficient C_D^n:

$$C_D^n = (\frac{u_*}{u_n})^2 = (\frac{\kappa}{b_n})^2. \tag{6}$$

In case of a neutrally stratified surface layer, u_n equals the actual wind, while for stable/unstable situations u_n is weaker/stronger. For given neutral wind at a level z and given Charnock parameter, drag coefficient C_D^n and roughness length z_0 can be found by inversion of (3-5). Usually, this solution is evaluated by iteration. Starting from some first guess value of u_* (for z_{10}, $u_* = u_{10}/25$ is a popular choice), an update of u_* is provided in (3-5), which is re-substituted into (3) until convergence is reached.

In case for an observation operator in data assimilation, where stress, roughness length or drag coefficient is to be estimated from an available (neutral) wind at lowest model level u_n, the necessity of fast and stable adjoint and tangent-linear code inhibits the usage of iteration. One example is the interpolation of lowest-level model wind ($\sim 10\mathrm{m}$) to buoy wind at observation height (typically 4 or $5\mathrm{m}$). Another application would be the calculation of model surface stress for comparison with observed scatterometer stress. Currently, in IFS, such calculations are based on the assumption of a constant roughness length of $z_0 = 1\mathrm{mm}$, which is typically one order of magnitude too high over the ocean.

For these applications, a direct, approximate relation between drag and neutral wind is highly desirable. This is the objective of this document. In Section (2) such a relation is presented. Section (3) summarizes results, and demonstrates the quality of the obtained expression for a typical situation.

2 Approximate solution

For a large domain in wind speed, (3) is dominated by either kinematic viscosity or Charnock. For this reason, first separate fits will be established for each regime. These two fits will then be combined into a proper fit for the entire wind domain. Fits will be provided for b_n, from which z_0 and C_D^n can be found analytically.

2.1 Kinematic viscosity

For sufficiently light wind, the first term in (3) will dominate. Combination of (3-5) implies the following implicit dimensionless equation for b_n:

$$b_n(e^{b_n} - 1) = R, \quad \text{where} \quad R = \frac{z}{\alpha_M \nu}(\kappa u_n). \tag{7}$$

Typically, $b_n \gg 1$, so (7) suggests that b_n is proportional to $\log R$. The following fit appears appropriate:

$$b_n^\nu = -1.47 + 0.93 \log R. \tag{8}$$

Evidence for this is provided in the left panel of Figure (1), which shows a very good match between the exact relation (7) (solid curve) and fit (8) (red dashed curve) over the typically encountered domain of R. This is illustrated by the three vertical grey lines, which indicate an extremely low ($z = 4\mathrm{m}$, $u_n = 0.1\mathrm{ms}^{-1}$), typical ($z = 10\mathrm{m}$, $u_n = 8\mathrm{ms}^{-1}$), and extremely high value ($z = 30\mathrm{m}$, $u_n = 50\mathrm{ms}^{-1}$) for R.

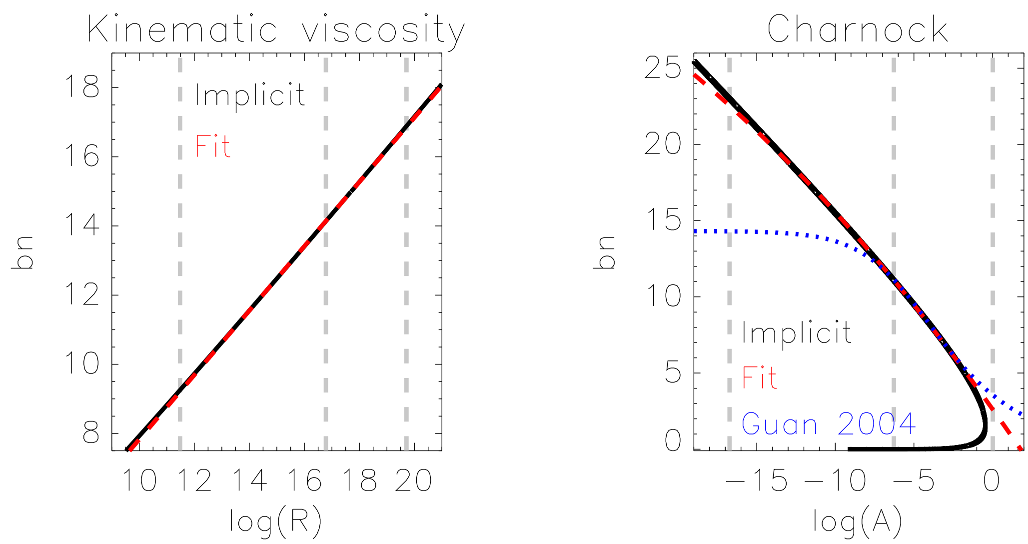

Figure 1: The relation between $b_n = \log(1+z/z_0)$ and neutral wind at height z in case the effect of kinematic viscosity (left panel) or Charnock (right panel) is dominant. R and A are defined in (7) and (9), respectively. Solid curves represent the exact relation, dashed red curves the fits (8) and (10), and the blue dotted curve relation (11) from Guan and Xie (2004).

2.2 Charnock

For sufficiently strong wind, the second term in (3) will dominate. Combination of (3-5) implies the following implicit dimensionless equation for b_n:

$$b_n^2/(e^{b_n}-1) = A, \quad \text{where} \quad A = \frac{\alpha_{\rm ch}}{gz}(\kappa u_n)^2, \tag{9}$$

which is proportional to the square of the Froude number $\tilde{u} = u_n/\sqrt{gz}$. Note that the function for b_n has a maximum $A_{\max} = 0.648..$ for $b_n = 1.593...$ Below this maximum, there are two solutions b_n for each A. This is illustrated by the black solid curve in the right panel of Figure (1). The smaller solution (where $b_n \sim A$) is valid when z is extremely close to the surface. The larger solution is valid for the more usual case when $z \gg z_0$. For that branch b_n seems proportional to $-\log A$. A proper fit, though, requires the inclusion of a higher order term as well:

$$b_n^\alpha = 2.65 - 1.44\log A - 0.015(\log A)^2. \tag{10}$$

The accuracy of this fit can be judged from the dashed red curve in the right panel of Figure (1). The agreement with the exact curve is good over the in practise encountered range. This, again, is marked by vertical grey dashed lines that indicate extremely low ($z = 30$m, $u_n = 0.1$ms^{-1}, $\alpha_{\rm ch} = 0.01$), typical ($z = 10$m, $u_n = 8$ms^{-1}, $\alpha_{\rm ch} = 0.018$), and extremely high ($z = 4$m, $u_n = 50$ms^{-1}, $\alpha_{\rm ch} = 0.1$) values for A. Note that this latter extreme value is slightly above $A_{\max}$. So, formally no exact solution exists, and the usual iteration of (3-5) will not converge. Although such situations should not occur, (10) will return a value for b_n. Hence, it is numerically more robust.

Previously, for C_D^n a dimensionless fit based on a Charnock relation had been established by Guan and Xie (2004):

$$C_D^{\rm Guan} = (0.78 + 4.7Y)\times 10^{-3}, \quad \text{where} \quad Y = \sqrt{\alpha_{\rm ch}}\tilde{u} = \sqrt{A}/\kappa. \tag{11}$$

For $\alpha_{\rm ch} = 0.0185$ it was shown to be consistent with a linear relation proposed by Wu (1980). Relation (11) is,

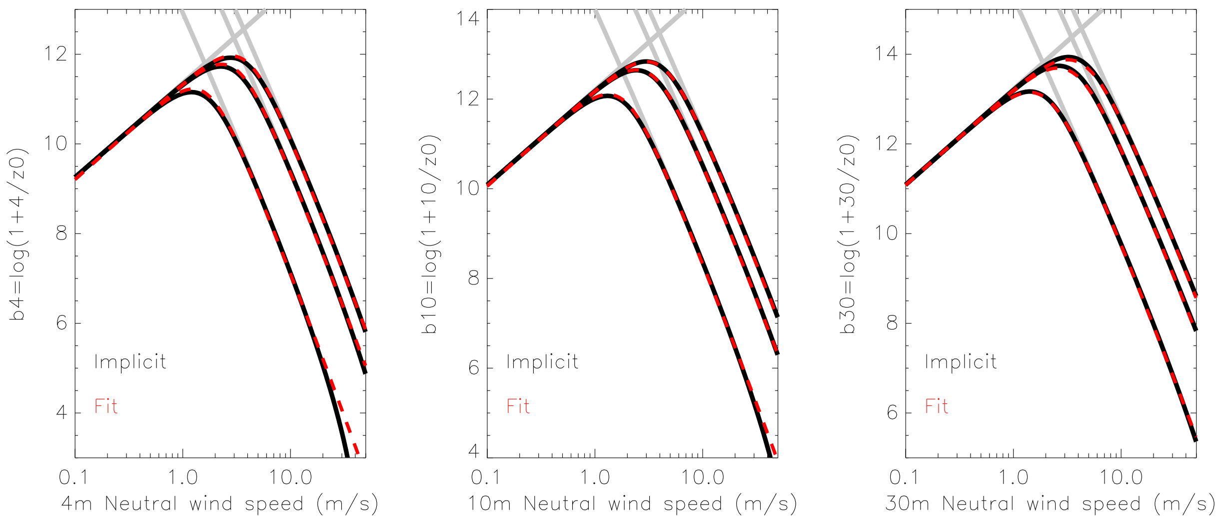

Figure 2: The relation between b_n and neutral wind at a height of 4m (left), 10m (middle) and 30m (right panel) and for a Charnock constant of 0.01 *(right),* 0.018 *(middle) and* 0.1 *(left curves) The solid black curves represent the iterative solution; the red dashed the fit, and the grey curves the branches for which either kinematic viscosity (left) or Charnock (right) would play a dominant role.*

after conversion to b_n, displayed in the right panel of Figure (1) as well (blue dotted curve). For a fair region, the agreement with exact implicit relation (9) is excellent as well.

2.3 Combination of both regimes

For cases of moderate wind speed, both terms in (3) will contribute. It appears that the following mix gives quite satisfactory results over the entire wind domain:

$$b_n^{\mathrm{fit}} = [(b_n^{\nu})^p + (b_n^{\alpha})^p]^{\frac{1}{p}} \quad \text{where} \quad p = -12. \tag{12}$$

The quality of this fit is demonstrated in Figure (2), which represents the situation for $z = 4$m (left), 10m (middle) and 30m (right panel), for $\alpha_{\mathrm{ch}} = 0.01$, 0.018 and 0.1. In this Figure, the exact relation is indicated by the black solid lines, fit (12) by the red dashed curves, while the branches (8) and (10) are represented by the grey solid curves. In the regime where either kinematic viscosity or Charnock dominates, the value of b_n for the other branch is so much larger that it does not contribute to the weighted average (12). Only where both effects contribute to z_0, a smooth, though rather fast transition between both branches occurs. The choice of value $p = -12$ appears to represent this transition well. As can be seen from Figure (2), fit (12) is able to describe the exact solution extremely well over a large range for level height, neutral wind speed, and Charnock parameter. Only for extreme values of α_{ch} and wind speed, a noticeable deviation occurs. However, this is in the region where no formal solution exists, and which should in practise not occur.

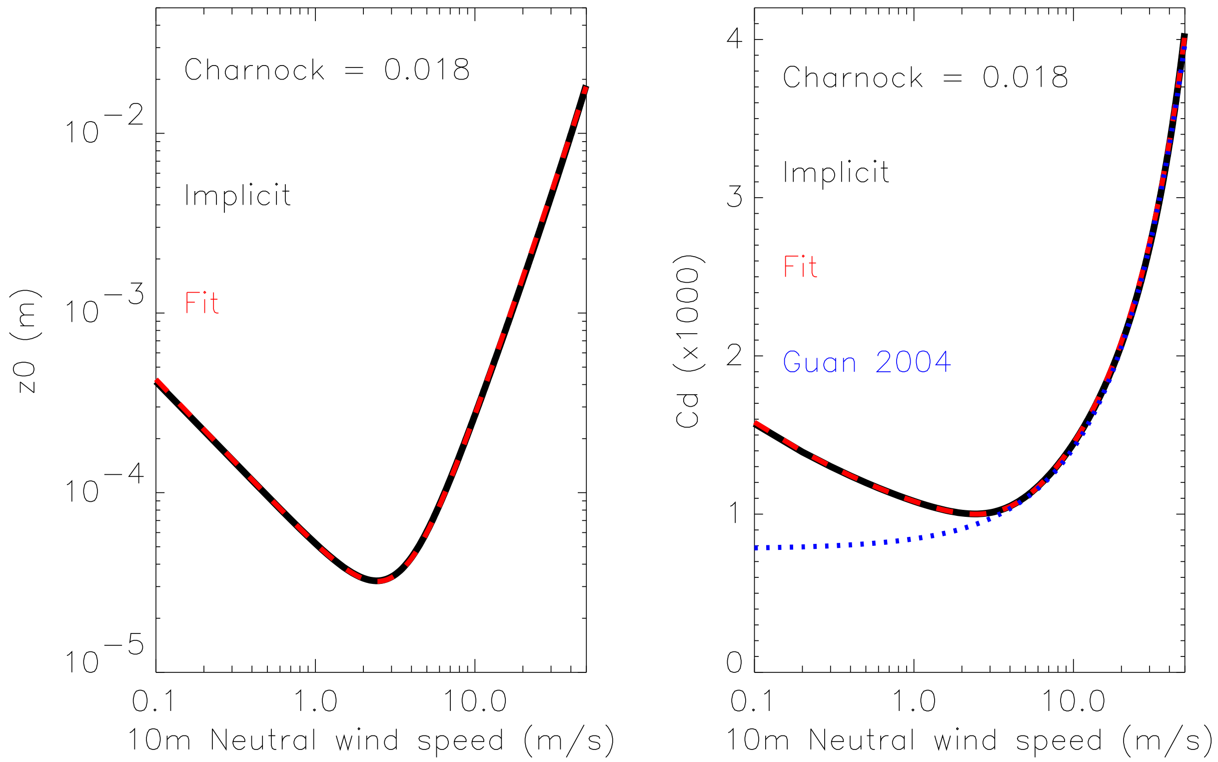

Figure 3: Roughness length z_0 (left) and neutral drag coefficient C_D^n (right) as function of 10m neutral wind speed for $\alpha_{\rm ch} = 0.018$. *Solid black curve is the solution obtained from iteration of (3-5), while the red dashed curves represent fit (13) and the blue dotted curve relation (11) from Guan and Xie (2004).*

3 Summary

The following relations provide an accurate fit for the behaviour of roughness length z_0 and drag coefficient C_D^n as function of neutral wind speed u_n at level height z, and Charnock parameter $\alpha_{\rm ch}$:

$$z_0^{\rm fit} = z/(\exp(b_n^{\rm fit}) - 1), \quad C_D^{n\,\rm fit} = (\frac{\kappa}{b_n^{\rm fit}})^2, \tag{13}$$

where

$$\begin{aligned} b_n^{\rm fit} &= [(b_n^{\nu})^p + (b_n^{\alpha})^p]^{\frac{1}{p}}, & p &= -12, & (14)\\ b_n^{\nu} &= -1.47 + 0.93 \log R, & R &= \frac{z}{\alpha_{\rm M}\nu}(\kappa u_n), & (15)\\ b_n^{\alpha} &= +2.65 - 1.44 \log A - 0.015 (\log A)^2, & A &= \frac{\alpha_{\rm ch}}{gz}(\kappa u_n)^2,. & (16) \end{aligned}$$

No iteration is required. The fit is entirely formulated in terms of the dimensionless quantities R and A, which besides physical quantities include the value of constants. For this reason the fit is not only valid over a wide range of level heights, wind speed and Charnock parameter, but also for alternative choices of constants (like κ and $\alpha_{\rm M}$).

In Figure (3) the quality of (16-16) for z_0 (left panel) and C_D^n (right panel) is demonstrated for 10m neutral wind at a Charnock parameter of 0.018. For the entire wind domain from $0.1 - 50 {\rm ms}^{-1}$, there is no noticeable difference from the implicit solution for (3-5). In the right panel, the drag coefficient as formulated by Guan and Xie

(2004) is displayed as well. From 3ms^{-1} and higher, the fit is very accurate as well. For low wind, this drag saturates, mainly because it does not take account of kinematic viscosity.

For the determination of z_0 (16-16) has been coded in a IFS (CY35R2) routine Z0SEA.F90 (where very low wind speed is capped to 0.1ms^{-1}). Its adjoint and tangent-linear versions are coded in Z0SEATL.F90 and Z0SEAAD.F90, respectively. Although these routines involve the derivative of roots and powers up to $p = -12$, the functional behaviour is very smooth, as can be seen from Figure (2). Currently, these routines are only used when scatterometer data is assimilated as equivalent neutral wind (which is not the default for CY36R2 and before).

Acknowledgements

I would like to thank Peter Janssen and Anton Beljaars for useful discussions and suggestions. The work presented in this document was partly funded by ESRIN (Project Ref. 22025/08/I-EC).

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